IU2

Modul Universal constants

Gravitational constants

In addition to his formulation of the law of motion ISSAC NEWTON'S, perhaps his greatest contribution to physics was the discovery of the general common law of gravitation. It describes the interaction between between two bodies, planets, or even smaller particles that causes a movement that can be described by the KEPLER'S Laws. The law was formulated in 1666 by NEWTON and 1687 was published as a chapter of his monumental work *Philosophiae Naturalis Principia Mathematica*.

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1.1 Preliminary Questions

- How is inertia defined?
- What is torsion?
- What is torque?
- What is the unit of the gravitational constant?
- How does the experiment of Cavendish function?
- What is harmonic motion?
- What is the corresponding equation of motion?
- What happens when a shaft is drilled through the center of the earth and a ball is tossed in?

1.2 Theory

1.2.1 Newton's law of gravity

KEPLER'S second law states that the force with which the gravitational interaction is associated with is a central force. This means that the force acts along a connection line between the centers of two interacting bodies. If we assume that the gravitational interaction is a general property, we must on the other hand consider the force F, which is associated with the interaction and is proportional to the "amount" of matter in any body, and the proportion of the corresponding masses m_1 and m_2. We can therefore write:

$$F = m_1 \cdot m_2 \cdot f(r) \tag{1.1}$$

It is difficult to determine the dependence of the force F on the distance r. Basically, the dependence is determined experimentally by the force between the masses m_1 and m_2 and is measured at different distances, whereby the relationship between F and r can finally be derived. Such experimental determination is indeed possible. However, it requires sensitive measuring equipment and relatively large patience. However, NEWTON had no such experimental possibilities. He recognized, motivated by KEPLER laws as the law of gravity had to be designed to:

> The gravitational interaction between two bodies, by a central attraction, is expressed that the masses of the bodies are directly proportional and the square of the distance between them is inversely proportional.

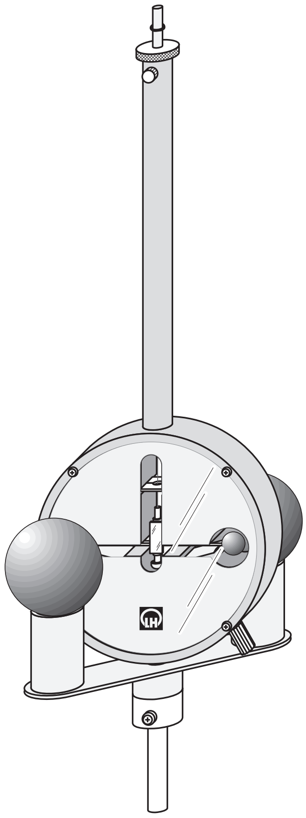
Or something more modern terms:

$$F = \gamma \cdot \frac{m_1 \cdot m_2}{r^2} \tag{1.2}$$

wherein γ is the proportionality or gravitational constant. However, with Eq. 1.2, the two interacting bodies are to be understood as point masses. For the description of planetary orbits, while the expansion of the planet is neglected, since these are small compared to the radii of the planetary orbits.

1.2.2 The Experiment of Cavendish

The core piece of the gravitational balance from Cavendish is on a thin torsion hairline cross-bar suspended horizontally and at each end at a distance d, a suspension point carries a small lead ball of mass m_2. These balls are big two lead balls tightened of mass m_1 according to Eq. (1.2). Although this force is less than 10^{-9}N, it can be detected with an extremely sensitive torsional balance. Motion observed the movement of the small lead balls and is measured via a laser pointer (see Fig. 1.1)



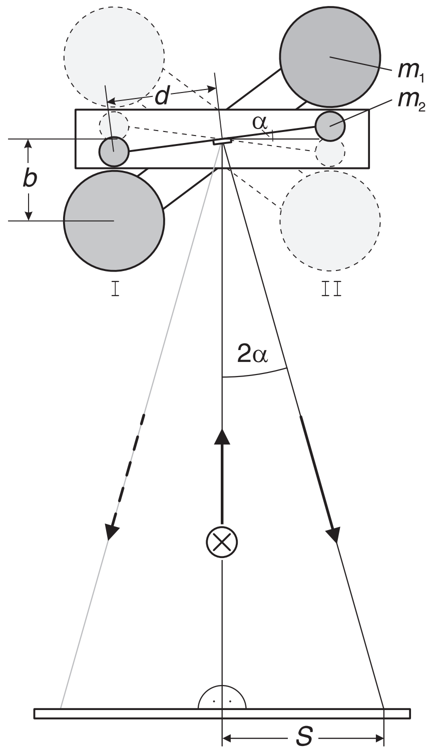


Figure 1.1: gravitation torsional balance from Cavendish (left) and schematic illustration of the laser pointer (right).

This is made by means of an illuminated concave mirror and is fixed rigidly to the crossbar of the torsional pendulum. From the chronological course of the movement, the mass m_1 and the geometry of the arrangement is determined on the basis of the gravitational constant from the following considerations in the next section.

1.2.3 Determination of the gravitational constant

The gravitational force between two lead balls of mass m_1 and m_2 at a distance b according to Eq. (1.2) is:

$$F_G = \gamma \frac{m_1 m_2}{b^2}, \tag{1.3}$$

when the large lead balls are in position I (see Fig. 1.1), thus acting on the torsion pendulum of the torque:

$$M_I = 2Fd = 2\gamma \frac{m_1 m_2}{b^2} d \tag{1.4}$$

This will be compensated by the restoring moment of the torsion wire, so that the pendulum assumes an equilibrium α_I.

By swirling the big balls in position II, it can now force about the symmetrical, so that a torque $M_{II} = -M_I$ is now effective and the damped oscillations of the pendulum can perform about the new equilibrium position α_{II}. The difference between the two rotary moments is given by:

$$D(\alpha_I - \alpha_{II}) = M_I - M_{II} = 2M_I \tag{1.5}$$

where the angular directional size D from the oscillation period T and the moment of inertia J of the torsional pendulum can be determined according to:

$$D = J\frac{4\pi^2}{T^2} \tag{1.6}$$

The moment of inertia is now in contact with the inertia of the two small balls:

$$J = 2m_2 d^2 \tag{1.7}$$

Thus, for Eq. (1.6):

$$D = m_2 d^2 \frac{8\pi^2}{T^2} \tag{1.8}$$

1.2.4 Measurement of the rotation angle α

In Fig. 1.1, the measurement of the rotation angle α is described with the help of the laser pointer. The illumination beam of the laser pointer is perpendicular here to the zero position of the torsional pendulum (the rest position without large lead balls). The laser pointer position for the zero is in line with the zero scale agreement. Between the rotation angle α, the laser pointer position S, and the distance L_0 between the scale and torsion, the connection is:

$$\tan(2\alpha) = \frac{S}{L_0} \tag{1.9}$$

for small angles α, respectively:

$$\alpha = \frac{S}{2L_0} \tag{1.10}$$

In Fig. 1.2, the concave mirror is illuminated below the horizontal angle β.

The laser pointer position O for the zero point of the torsional pendulum has the distance L_1 to the receiving point N of the normal and the distance to the concave mirror:

$$L = \sqrt{L_0^2 + L_1^2}. \tag{1.11}$$

For rotation of the torsion pendulum at the angle α from the zero position, the relationship is found as:

$$S' = L\tan(2\alpha) \tag{1.12}$$

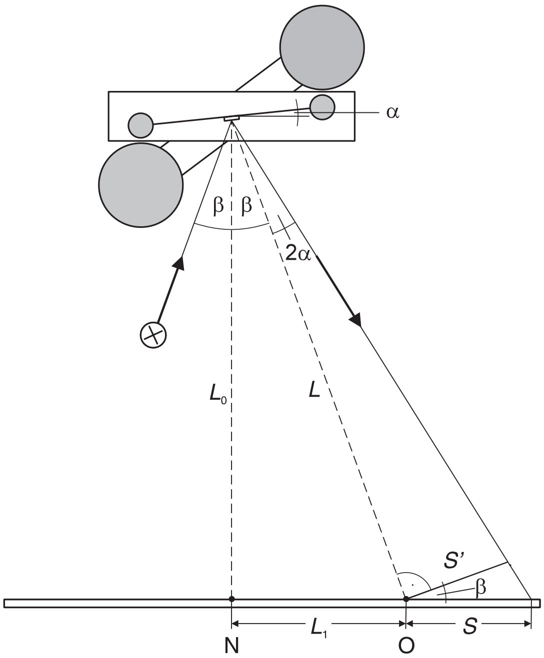


Figure 1.2: Scheme for determining the deflection of the laser pointer.

and

$$\frac{S'}{S} = \frac{\sin(90^\circ - \beta - 2\alpha)}{\sin(90^\circ + 2\alpha)} = \cos(\beta) - \tan(2\alpha)\sin(\beta) \tag{1.13}$$

The angle α is in any case very small (it is at most 1.5°), the dimensions of the gravitational torsion balance can not be illumination at the angle β above 30°. Therefore, the approximation:

$$\frac{S'}{S} = \cos(\beta) = \frac{L_0}{L} \tag{1.14}$$

is allowed. With the additional approximation $\tan(2\alpha) \approx 2\alpha$, thus follows the total :

$$\alpha = \frac{S}{2}\frac{L_0}{L_0^2 + L_1^2} \tag{1.15}$$

This equation (1.15) is subjected to a systematic error $1 - 2\%$, by the calculation of the difference between the two positions of equilibrium $\alpha_I - \alpha_{II}$ and this systematic error will all but almost be completely compensated.
For the special case of illumination with β at small angles and $L_0 \gg L_1$, one obtains from Eq. (1.15) the already derived equation (1.10).
Eq. (1.15) is also valid when the illumination beam is tilted up or down. It depends also in this case, on the horizontal reading scale and can be disregarded from the laser pointer.
The zero position of the torsion pendulum, the point O in Fig. 1.2, is usually unknown in front of the enforcement. To determine L_1, one measures, in good approximation, the distance between the normal point N and the laser pointer position for the equilibrium position I. This approximation is permitted because of $|\alpha| \ll 1$. When oblique illumination is not in the direction of the concave mirror, i.e. for $\beta \ll 1$, $L_1 = 0$ can be accepted.

1.2.5 Counter torque of the second lead ball

In addition to the torque by the attractive force F of the opposing large lead ball (distance b) is a counter torque through the attraction force F_2 of the respective distant ball (distance d') is generated (see Fig. 1.3).

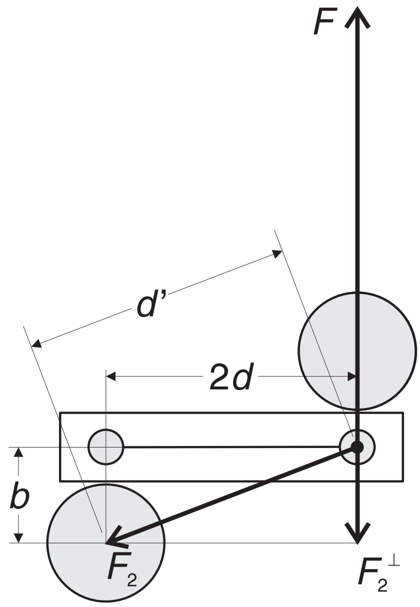


Figure 1.3: Scheme for calculating the counter torque by the "second" lead ball.

For the torque M_I, it gives exactly as in Eq. (1.4):

$$M_I = 2(F + F_2^{\perp})d \tag{1.16}$$

where

$$F_2^{\perp} = -F_2 \frac{b}{d'} \tag{1.17}$$

perpendicular to the cross-bar component of the force:

$$F_2 = F \frac{b^2}{d'^2} \tag{1.18}$$

(see Eq. (1.4)). Therefore, the gravitational constant γ with the correction factor is:

$$K = \frac{F}{F + F_2^{\perp}} = \frac{1}{1 - \frac{b^3}{d'^3}} \tag{1.19}$$

greater. With

$$d' = \sqrt{(2d)^2 + b^2} \tag{1.20}$$

with the numerical value K calculated.

1.3 Experiment

1.3.1 Experimental Data

Size	Value
Mass of the large ball	1500 g
Distance of the small balls from the point of suspension	4.94 cm
Distance of the small balls from the large ball	4.85 cm
Distance of the suspension point of the wire to the scale	277 cm

1.3.2 Experimental Setup and Adjustment

- For at least two hours ensure there is no vibration before making the arrangement blank so that the pendulum may swing into the equilibrium position.
- Control the stability of the zero point.
- Observer zero fluctuations for at least 10 min.

1.3.3 Measurements

Important: While switching the ball carrier, absolutely avoid shocks of the housing by hitting the lead balls.

- Switch on the laser and record the start position of the laser pointer.
- Put the lead balls carefully rotated from position I to position II and start the stop watch.
- Read the position of the laser pointer on the scale for 30 min at least every 30 seconds, until the oscillation has subsided.
- Waiting about 60-90 minutes until the system is in the equilibrium state again.
- Write down again the starting position of the laser pointer.
- Swing the lead ball from position II back into position I and repeat the measurement.
- Put the lead ball aligned perpendicular to the housing, remove the ball, and measure the position of the zero position.
- Put it back in position I gently and turn off the laser.

1.3.4 Tasks for Evaluation

- Determine the period and the equilibrium position of the two series of measurements by a performing a fit of the corresponding function on the data.
- Determine the gravitational constant, the correction factor, and the corrected gravitational constant.
- Make a complete error calculation.
- Compare the literature value using your result.