

IU1

Modul Universal Constants

Gravitational Acceleration

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Versuch IU1 - Gravitational Acceleration

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1.1 Preliminary Questions

- How can one determine the gravitational acceleration with a mathematical pendulum?
- How does the period of oscillation of the mathematical pendulum depend on the acceleration due to gravity? Which approximation is necessary?
- How does the gravitational acceleration depend on the fall time and the fall height?
- Give reasons why the gravitational acceleration is location-dependent.

1.2 Theory

1.2.1 The Free-Fall Experiment

The Newtonian equation of motion for the free fall of a body in a gravitational field reads:

$$F_G = m \frac{d^2s}{dt^2} \quad \text{d.h.} \quad \frac{d^2s}{dt^2} = g \quad (1.1)$$

The only acting force is the weight $F_G = mg$ of the body. From the differential equation $d^2s/dt^2 = g$, it is easy to estimate the integration between gravitational acceleration g , fall time T , and drop distance L . In reality, the case in the air, of course, is not entirely free. The air resistance acts against the acceleration. Therefore, the above equation of motion receives an additional term:

$$\frac{d^2s}{dt^2} = \frac{dv}{dt} = g - f(v) \quad (1.2)$$

The air resistance increases linearly with the velocity for relatively low velocity and quadratic for high velocities. For our experiment, we can speak higher velocities and it is:

$$f(v) = \beta v^2 \quad (1.3)$$

We want to, in the following equation of free fall motion, taking into account air resistance,

$$\frac{d^2s}{dt^2} = \frac{dv}{dt} = g - \beta v^2$$

solve. We first introduce a separation of variables, which can be easily integrated.

$$dt = \frac{dv}{g - \beta v^2} \quad \Rightarrow \quad \int_0^{t_E} dt = \int_0^{v_E} \frac{dv}{g - \beta v^2}$$

By substituting $x^2 = \frac{\beta v^2}{g}$, we get:

$$t_E = \frac{1}{\sqrt{g\beta}} \int_0^{\sqrt{\frac{\beta}{g}} v_E} \frac{dx}{1 - x^2} = \frac{1}{\sqrt{g\beta}} \tanh^{-1}\left(\sqrt{\frac{\beta}{g}} v_E\right)$$

We generalize this result for all times and velocities and solving for v , we obtain:

$$v(t) = \sqrt{\frac{g}{\beta}} \tanh(t\sqrt{g\beta})$$

From this, we easily obtain the time t in the distance s traveled by integrating over time (substitution $x = t' \sqrt{g\beta}$)

$$\begin{aligned} s(t) &= \int_0^t v(t') dt' = \sqrt{\frac{g}{\beta}} \int_0^t \tanh(t' \sqrt{g\beta}) dt' \\ &= \frac{1}{\beta} \int_0^{t\sqrt{g\beta}} \tanh x dx = \frac{1}{\beta} \ln \cosh(t\sqrt{g\beta}) \end{aligned}$$

We have obtained the result:

$$\boxed{s(t) = \frac{1}{\beta} \ln \cosh(t\sqrt{g\beta})} \quad (1.4)$$

For small arguments $t\sqrt{g\beta}$, we obtain the free fall through approximations:

$$\begin{aligned} \cosh x &\approx 1 + \frac{1}{2}x^2, & \ln(1+y) &\approx y \\ \Rightarrow s(t) &\approx \frac{1}{\beta} \cdot \frac{1}{2}t^2 g\beta = \frac{1}{2}gt^2 \end{aligned}$$

1.3 Experiment

1.3.1 Equipment

Component	Number
Steel ball \varnothing 16mm	1
Holding magnet	1
STE-Si-Diode N 4007	1
Fork light barrier	2
6-pin connection cable	2
Digital counters	1
Foot stand	1
Support rod 150cm	1
Support rod 25cm	1
Sleeve	1
Height scale 1m	1
Base	1
Fishing Line	1
Last piece	6
Experiment cables	1

1.3.2 Experimental Setup

The experimental setup is shown in Figure 1.1 and the description of the holding magnet is shown in Figure 1.2.

- Attach the fork sensors with the red LED pointing upwards and connect the 6-pin connecting cables to the input E and F of the digital counter.

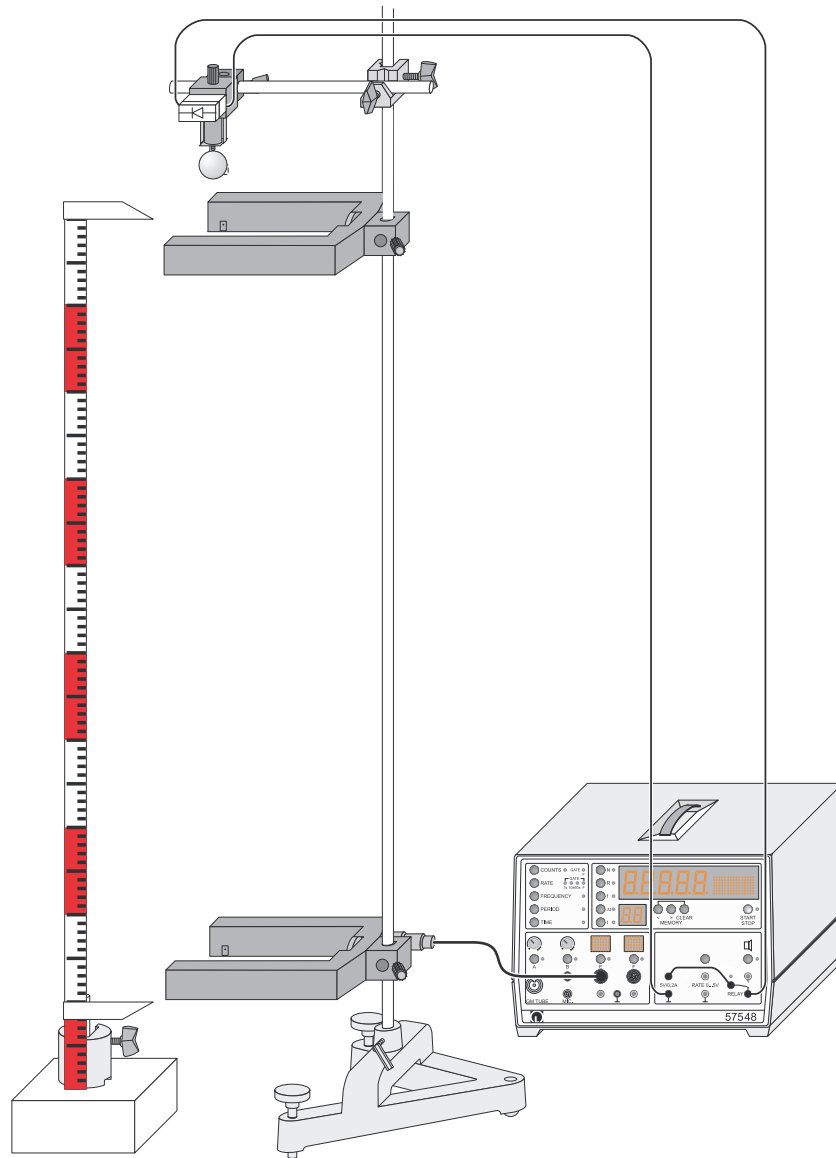


Figure 1.1: Experimental Setup for measuring the fall time between the holding magnet and the two fork light barriers.

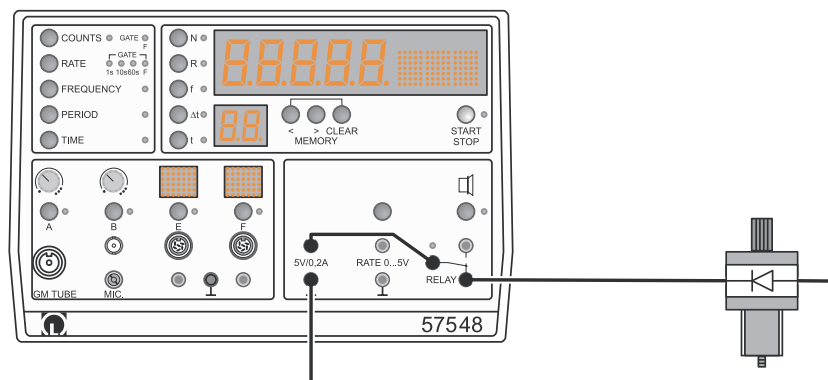


Figure 1.2: Connection of the holding magnet and the digital counter.

- Mount the short stand rod with the sleeve and fasten the holding magnet underneath.
- Connect the positive pole of 5V output via the relay of the digital counter with a jack and connect it with the second sleeve of the holding magnet and STE Si-diode on the plug input of the holding magnet (see Figure 1.2).
- Turn on the digital counter switch and press the time button (for case in s).
- Press the *E* and *F* key repeatedly until the display $\downarrow \square$ appears.
- Hang the steel ball on the magnet.
- Align the upper fork light barrier and holding magnet with the steel ball precisely such that the light barrier is discontinuous by the lower edge of the steel ball (observe with the red LED).
- Height scale up by hands and upper pointer as zero of the path to the com-trailing edge align the upper fork light barrier.

1.3.3 Measurements

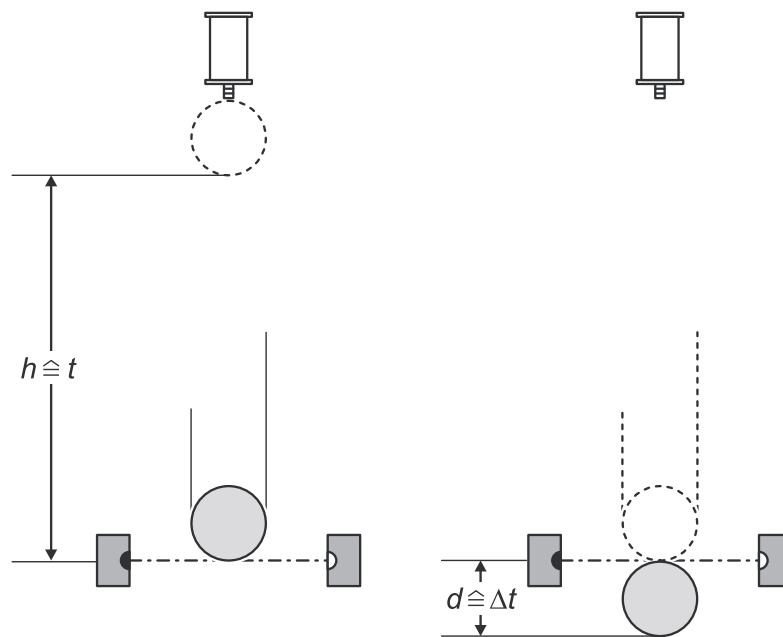


Figure 1.3: Schematic diagram of measuring the fall time and the black out time Δt of a falling ball with the light barrier.

- Position the upper fork light barrier to drop a distance of 10cm and the lower fork light barrier for a fall distance of 40cm.
- Unscrew the milled screw of the holding magnet and pass the fishing line and at the lower end attach the last piece.
- Adjust the arrangement so that the two light barriers are discontinuous through the fishing line.

- Remove the fishing line and screw in the milled screw.
- Attach the steel ball and turn back the milling screw until the steel ball is just touching (mark on the milling screw to reproduce the setting again).
- Attach the steel ball and start the measurement with key **START STOP**.
- Press the **START STOP** button again as soon as the ball has been dropped.
- Record the fall time t in s .
- Press the button Δt twice and blackout time (see Figure 1.3) and read Δt in ms and note.
- In memory recall, press the “<” button and read the second pair of values “ Δt ” and Δt and note.
- Repeat the measurement of the fall time and blackout time 10 times.
- Push the light barrier under a distance of 90cm and check if the adjustment of the fishing line of the assembly is necessary
- Repeat the measurements for 5 more fall distances (varying only the drop height of the second light barrier between 40cm and 90cm. The first light barrier remains constant at 10cm).

1.3.4 Tasks for Evaluation

- Calculate the mean, standard deviation, and the standard deviation of the mean for all measured variables.
- Calculate the distance as a function of time from the equation of motion for the free fall of the drop.
- Determine the gravitational constant using this formula from the measurements for each case, the amount and value with the corresponding errors. Fill the experimental g with error bars as a function of the fall height. Compare the theory value with that of the precision measurements and discuss the various possible systematic deviations.
- Calculate the gravitational acceleration of all the values and the mean of the corresponding error.
- Create a graph of the drop distance to the first light barrier, the second light barrier, and the drop distance from the first to the second light barrier as a function of the fall time.
- Create a graph of the instantaneous velocity when passing through the two light barriers.
- Create a graph of the three previous fall distances as a function of the square of the fall time and determine the resulting gravitational acceleration.
- Compare the gravitational acceleration data obtained from the two methods.

1.4 Literature

- W. Greiner, “*Klassische Mechanik*”, Verlag Harri Deutsch
- D. Meschede, “*Gerthsen Physik*”, Springer Verlag