IU1

Modul Universal Constants

Gravitational Acceleration

The goal of this experiment is to determine the gravitational acceleration g from the falling time of a body during the (fast) free fall through the air.

Versuch IU1 - Gravitational Acceleration

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1.1 Preliminary Questions

- How does the gravitational acceleration depend on the fall time and the fall height?
- Why is the gravitational acceleration is location-dependent?
- What will we find with this experiment, if we measure the gravitational acceleration
  a) here in Basel (Uferstrasse 90)?
  b) on a plane at an altitude of 10 km and a velocity of 900 km/h?
  c) on the International Space Station ISS with altitude 385 km and a constant velocity of 7.66 km/s?
- How does a light barrier work? Therefore, what part of the experimental setup do we need to be mindful about?

1.2 Theory

1.2.1 The Free-Fall Experiment

The Newtonian equation of motion for the free fall of a body in a gravitational field reads:

$$F_G = m\frac{d^2s}{dt^2} \qquad \text{d.h.} \qquad \frac{d^2s}{dt^2} = g \tag{1.1}$$

The only acting force is the weight $F_G = mg$ of the body. From the differential equation $d^2s/dt^2 = g$, it is easy to estimate the integration between gravitational acceleration g, fall time T, and drop distance L. In reality, the case in the air, of course, is not entirely free. The air resistance acts against the acceleration. Therefore, the above equation of motion receives an additional term:

$$\frac{d^2s}{dt^2} = \frac{dv}{dt} = g - f(v) \tag{1.2}$$

The air resistance increases linearly with the velocity for relatively low velocity and quadratic for high velocities. For our experiment, we can of speak higher velocities and it is:

$$f(v) = \beta v^2 \tag{1.3}$$

We want to, in the following equation of free fall motion, taking into account air resistance,

$$\frac{d^2s}{dt^2} = \frac{dv}{dt} = g - \beta v^2$$

solve. We first introduce a separation of variables, which can be easily integrated.

$$dt = \frac{dv}{g - \beta v^2} \qquad \Rightarrow \qquad \int_0^{t_E} dt = \int_0^{v_E} \frac{dv}{g - \beta v^2}$$

By substituting $x^2 = \frac{\beta v^2}{g}$, we get:

$$t_E = \frac{1}{\sqrt{g\beta}} \int_0^{\sqrt{\frac{\beta}{g}}v_E} \frac{dx}{1 - x^2} = \frac{1}{\sqrt{g\beta}} \tanh^{-1}\left(\sqrt{\frac{\beta}{g}}v_E\right)$$

We generalize this result for all times and velocities and solving for v, we obtain:

$$v(t) = \sqrt{\frac{g}{\beta}} \tanh\left(t\sqrt{g\beta}\right)$$

From this, we easily obtain the time t in the distance s traveled by integrating over time (substitution $x = t'\sqrt{g\beta}$)

$$\begin{aligned} s(t) &= \int_0^t v(t')\,dt' = \sqrt{\frac{g}{\beta}} \int_0^t \tanh\left(t'\sqrt{g\beta}\right)\,dt' \\ &= \frac{1}{\beta} \int_0^{t\sqrt{g\beta}} \tanh x\,dx = \frac{1}{\beta} \ln\cosh\left(t\sqrt{g\beta}\right) \end{aligned}$$

We have obtained the result:

$$\boxed{s(t) = \frac{1}{\beta} \ln\cosh\left(t\sqrt{g\beta}\right)} \tag{1.4}$$

For small arguments $t\sqrt{g\beta}$, we obtain the free fall through approximations:

$$\cosh x \approx 1 + \frac{1}{2}x^2, \qquad \ln(1+y) \approx y$$

$$\Rightarrow \qquad s(t) \approx \frac{1}{\beta} \cdot \frac{1}{2} t^2 g\beta = \frac{1}{2} g t^2$$

1.3 Experiment

1.3.1 Equipment

Component	Number
Steel ball ∅ 16 mm	1
Holding magnet	1
STE-Si-Diode N 4007	1
Fork light barrier	2
6-pin connection cable	2
Digital counters	1
Foot stand	1
Support rod 150 cm	1
Support rod 25 cm	1
Sleeve	1
Height scale 1 m	1
Base	1
Fishing Line	1
Last piece	1
Experiment cables	1

1.3.2 Experimental Setup

The experimental setup is shown in Figure 1.1 and the description of the holding magnet is shown in Figure 1.2.

- Attach the fork sensors with the red LED pointing upwards and connect the 6-pin connecting cables to the input *E* and *F* of the digital counter.
- Mount the short stand rod with the sleeve and fasten the holding magnet underneath.
- Connect the positive pole of 5V output via the relay of the digital counter with a jack and connect it with the second sleeve of the holding magnet and STE Si-diode on the plug input of the holding magnet (see Figure 1.2).
- Turn on the digital counter switch and press the time button (for fall time in s).
- Press the *E* and *F* key repeatedly until the display ↓⊓ appears.
- Hang the steel ball on the magnet.
- Align the upper fork light barrier and hanging steel ball precisely such that the light barrier is blocked by the lower edge of the steel ball (observe with the red LED).
- Now put the upper pointer of the height scale on the height of lower border of the upper light barrier as a ′zero point′.

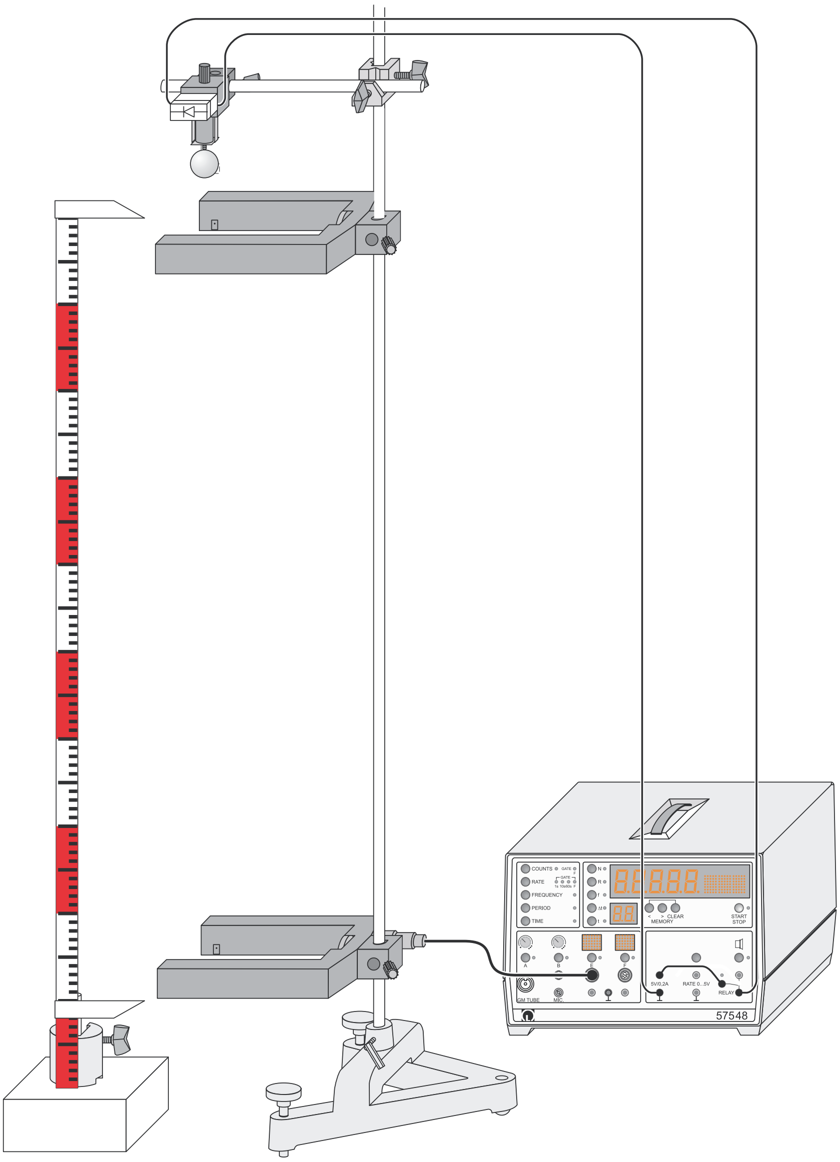

Figure 1.1: Experimental Setup for measuring the fall time between the holding magnet and the two fork light barriers.

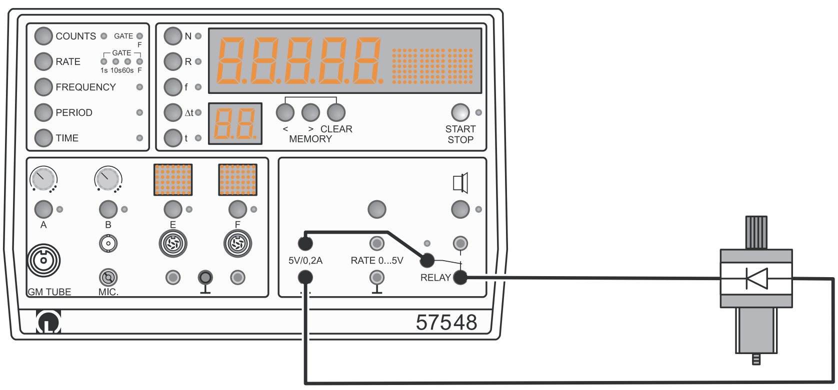

Figure 1.2: Connection of the holding magnet and the digital counter.

1.3.3 Measurements

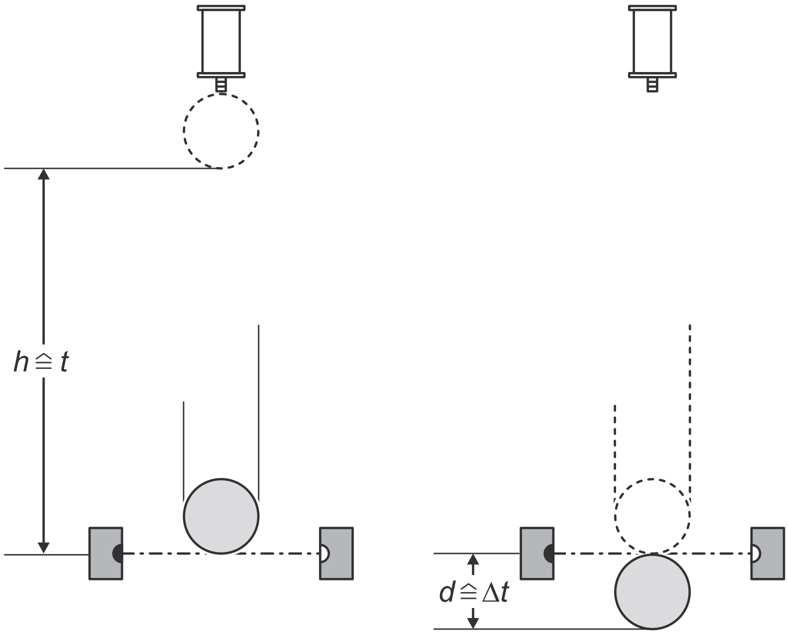

Figure 1.3: Schematic diagram of measuring the fall time and the black out time Δt of a falling ball with the light barrier.

- Position the upper light barrier to a fall distance of 10 cm (below the ’zero point’) and the lower fork light barrier to a fall distance of 40 cm.
- Detach the steel ball and use the fishing line to adjust the arrangement so that the two light barriers are blocked by the fishing line.
- Attach the steel ball and turn the milling screw such that the steel ball is just staying up.
- Attach the steel ball and start the measurement with key **START STOP**.
- Press the **START STOP** button again as soon as the ball has been dropped.
- Record the fall time t in s for both light barriers (use E and F keys to switch between them).
- Press the button Δt twice (see Figure 1.3) to read the blackout times Δt in ms and note again for both light barriers.
- Repeat the measurement of the fall time and blackout time 10 times.
- Put the lower light barrier to a fall distance of 50 cm and use the fishing line to make sure the light barriers are adjusted in line.
- Repeat the measurements for 4 more fall heights (varying only the fall height of the second light to 60, 70, 80 and 90 cm. The first light barrier remains constant at 10 cm).

1.3.4 Tasks for Evaluation

Numerical

- For every fall height for the ten measurements, calculate the mean value, the standard deviation and the standard deviation of the mean value. Repeat for the blackout times.
- With the equation of motion for the free-fall, calculate the fall height as a function of time.
- With this formula, calculate the gravitational acceleration g for every fall height and estimate the errors. Make a plot of the calculated g as a function of the fall height (with error bars).
- From these values for g, calculate the mean value and the associated errors.

Method 1

- Make a plot of the fall height to the second light barrier and from the first to the second light barrier as a function of time. Determine the gravitational acceleration with a fit (explain briefly how) and calculate the errors.
- This can also be achieved, if you plot the fall height as a function of the time squared (for the two cases as before). Determine the gravitational acceleration with a fit (explain briefly how) and calculate the errors.
- Compare the two results. What do you notice?

Method 2

- Calculate (with the blackout times) the momentary velocity at both light barriers for each fall height.
- Make a plot of the momentary velocity as a function of the fall height (only for the second light barrier). Determine the gravitational acceleration with a fit (explain briefly how) and calculate the errors.

Discussion

- Compare the obtained values for the gravitational acceleration with each other and with the literature value. Discuss the different possible systematic errors.
- Is the amount of measurements per fall height reasonable? Justify!

1.4 Literature

- W. Greiner, "*Klassische Mechanik*", Verlag Harri Deutsch
- D. Meschede, "*Gerthsen Physik*", Springer Verlag