



IO4

Modul Optics

Polarisation and Saccharimetry

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1.1 Preliminary Questions

- What is the general, mathematically, representation of light waves in physics? Name all occurring quantities.
- What is polarized light and which types of polarized light can be differed?
- How can one produced those types of polarized light?
- Light is electromagnetic radiation. What is the relation between the electric and the magnetic field? How do those fields change in case of polarization?
- Does polarization influence the energy or rather the momentum, transferred by light?
- What do the terms transversal and longitudinal wave refer to? Name three example for each of them. Can you polarize both types of waves?

1.2 Theory

1.2.1 Mathematical Description of electromagnetic Waves

Light is also known as electromagnetic radiation. Electromagnetic radiation is in principle described by TRANSVERSAL WAVES, meaning the oscillation is occurs perpendicular to the direction of propagation. The opposite of transversal waves are so called LONGITUDINAL WAVES, where the oscillation occurs parallel to the direction of propagation¹.

Electromagnetic radiation can be described by the vector \vec{E} , which represents the electric field, and the vector \vec{B} , representing the magnetic flux density (often just called magnetic field, which isn't fully correct). Both vectors, \vec{E} and \vec{B} , are perpendicular to the direction of propagation and to each other.

A light wave can be described by the following equation:

$$\psi(\vec{r}, t, \omega) = A \cdot e^{i(\vec{k}\vec{r} - \omega t + \Phi)} \quad (1.1)$$

This is a general form of a plane wave, which represents the solution to the HOMOGENEOUS WAVE EQUATION. Or more precisely, it is a homogeneous, linear differential equation of first order. From mathematics we know, that arbitrary linear combinations of solutions of linear equation are also solutions of the same equation. Therefore, linear combinations of plane waves are still a valid solution of this equation. From mathematical property results the *superposition principle* for waves.

Hereafter, we will concentrate on the real part of \vec{E} and \vec{B} , in addition we will assume, that the electric and magnetic field do only have a x- and a y- component and the propagation occurs in z-direction. Since \vec{B} behaves analogously to \vec{E} , it is sufficient to only consider \vec{E} . Therefore:

$$\vec{E}_x(z, t) = E_{0,x} \cdot \cos(k_z z - \omega t) \cdot \hat{e}_x \quad (1.2)$$

$$\vec{E}_y(z, t) = E_{0,y} \cdot \cos(k_z z - \omega t + \Phi) \cdot \hat{e}_y \quad (1.3)$$

$$|\vec{E}_0| = \frac{1}{c} \cdot |\vec{B}_0| \quad (1.4)$$

¹A prime example are sound waves, where air molecules oscillate back and forth

Where $\vec{E}_{0,i}$ describes the respective component of the amplitude \vec{E}_0 , ω the angular frequency, k_z is the z-component of the wave vector \vec{k} and Φ is the phase of the plane wave. Angular frequency and wave vector have the following relation to each other:

$$\frac{\omega}{c} = |\vec{k}| = \frac{2\pi}{\lambda} \quad (1.5)$$

Where c is the speed of light and λ is its WAVELENGTH.

The relation, stated in equation 1.4, between amplitude of the electric and the magnetic field applies generally.

1.2.2 Polarized electromagnetic Radiation

We covered the mathematical basics to describe electromagnetic radiation, in the last section and are now moving on to the phenomenon of polarization. Since the equations for 1.2 and 1.3 are representing two components of a vector, they can be added into one resulting vector. We assume, both components of the electric field are oscillating in phase, this implies $\Phi = 0$

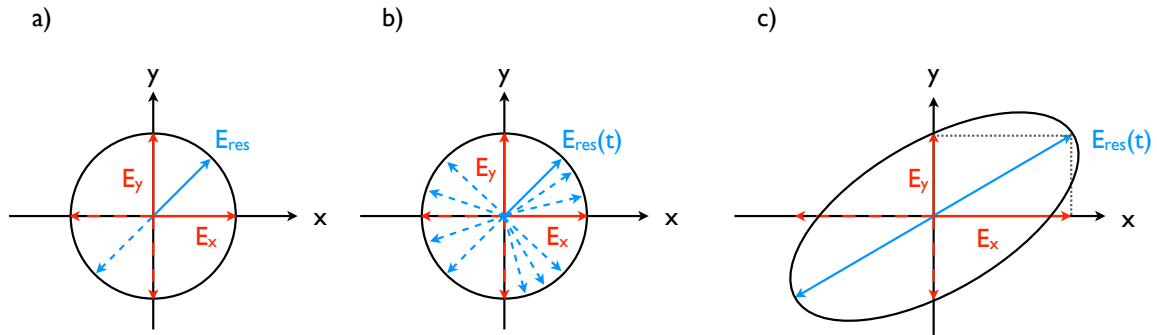


Figure 1.1: Schematic display of a) linear polarization, b) circular polarization and c) elliptical polarization.

or $\Phi = 2\pi n$, $n \in \mathbb{N}$. When adding up both vectors, we get:

$$\vec{E} = \vec{E}_x(z, t) + \vec{E}_y(z, t) = (E_{0,x} \cdot \hat{e}_x + E_{0,y} \cdot \hat{e}_y) \cdot \cos(k_z z - \omega t) \quad (1.6)$$

as a result (see Fig. 1.1 a). The initial components of the electric field are displayed in red, whereas the resulting field is shown in blue. The amplitude $(E_{0,x} \cdot \hat{e}_x + E_{0,y} \cdot \hat{e}_y)$ is constant and always points in direction $\hat{e}_x + \hat{e}_y$. This amplitude now modulated with an cosine only, meaning the resulting electric field is oscillating between the maxima $\pm (E_{0,x} \cdot \hat{e}_x + E_{0,y} \cdot \hat{e}_y)$. With time, all possible values between both maxima occur, but the direction stays the same (in this example $\hat{e}_x + \hat{e}_y$). This is called LINEAR POLARIZED LIGHT.

Now, let's assume, there is a phase shift of e.g. $\pm\pi$ between both components of the electric field. This phase shift causes vector \vec{E}_y to turn 180° , referring to Fig. 1.1, it now points downwards. So, when adding both vectors \vec{E}_x and \vec{E}_y , the resulting field points in opposite direction. This result is analogue to the previous example, but with a rotation of the polarization plane by 90° . Through similar consideration, one can easily realize that every direction of polarization can be produced.

Next, we have a look at the case where both components have the same amplitude, $E_{0,x} = E_{0,y} = E_0$, but with a phase shift of $\Phi = \pm\pi/2 \pm 2\pi n$, $n \in \mathbb{N}$. In this special case equation 1.6 can be rewritten as:

$$\vec{E}(z, t) = E_0 [\cos(k_z z - \omega t) \cdot \hat{e}_x + \sin(k_z z - \omega t) \cdot \hat{e}_y] \quad (1.7)$$

This situation can be seen in Fig. 1.1 b). It yields to a scalar amplitude E_0 with an altering direction. Rather, the polarization direction rotates as a function of time on the illustrated circle, which is the sum of all possible resulting electric fields. This fact is shown in Fig. 1.1 b) by the additional, dashed, blue arrows. In this case one speaks of CIRCULAR POLARIZED LIGHT.

Obviously, circularly polarized light can always be thought of as a composition of two mutually perpendicular linearly polarized components. Accordingly, circularly polarized light can be generated by the superposition of two linearly polarized components, provided both have the same amplitude (or intensity) and a phase shift of exactly $\Phi = \pm\pi/2 \pm 2\pi n$, $n \in \mathbb{N}$. Furthermore, one has to distinguish between right - handed (+) and left - turning (-) circular polarized light.

Finally, a third case has to be considered. This is de facto the most general case, and thus linear and circularly polarized ones can be regarded as special cases of this general case. In general, both components of the electric field will have different amplitudes, i.e., $E_{0,x} \neq E_{0,y}$, and they will have an arbitrary phase shift Φ to each other. Some algebraic transformations show that in this case the resulting electric field describes an ellipse as a function of time, as shown in Fig. 1.1 c).

1.2.3 Experimental Generation of Polarized elm. Radiation

Natural light emitted by the sun, for example, is generated by excited atoms emitting energy in the form of electromagnetic radiation.² Each excited atom emits polarized light for about 10^{-8} s. The enormous number of excited atoms, however, means that a part of the atoms just begin to emit a polarized wave train, whereas others have already begun earlier to emit or are just at the end of an emission phase. Hence, natural light consists of a multiplicity of different polarizations which occur, as a function of time, randomly. Therefore, it is often said that natural light is unpolarized. But, in fact this is not entirely correct as should be apparent from the discussion above. But the experiments carried out here require a well-defined polarization, as discussed in the previous section. Natural light, however, is not suitable because of its perfectly arbitrary and arbitrarily fluctuating composition of various polarizations.

The simplest method of producing linearly polarized light is by means of a so-called polarizing film. This is a grid of electrically conductive material. If electromagnetic radiation is applied to this grating, and if the wavelength is less than the distance between the parallel grating strings, the light will be polarized perpendicular to the grating strands. The reason for this is that the components of the electric field, which are parallel to the lattice rods, cause the electrons to move into the lattice lattice, the electrons are generated by the electrons, and the energy is lost. Thus, energy is transferred from the electric field to the lattice material, so the transmission for the components of the electric field perpendicular to the lattice states is zero. Analyzer films, as used in the experiment, work according to the same principle.

The linearly polarized light thus produced can now be obtained e.g. can be used to produce circularly polarized light, as previously discussed.

Furthermore, linearly polarized light is generated by REFLECTION of non-polarized light within the so-called BREWSTER ANGLE (see literature). In addition there is a certain number of minerals, e.g. Calcite, which have a polarizing effect. Sometimes, such crystals are also used in

²The light arriving on earth, emitted by the sun, comes from the approx. 5000 K hot, lowest atmosphere layer of the Sun, which is called Photosphere.

polarizers/analyzers where they are present in a transparent gel.

Circularly polarized or elliptically polarized light is produced e.g. in the so-called BIREFRINGENCE (see references), provided the incident light was already linearly polarized.

1.2.4 Optically active Substances and Saccharimetry

In this experiment, the polarization of light is examined by means of an interesting effect, which is referred to as OPTICAL ACTIVITY. It is evident that some substances are able to rotate the polarization plane of light. Of particular interest is this effect in chemistry or biochemistry. So-called enantiomers, are molecules which are completely identical in all their chemical and physical properties, except in their optical activity. In fact, these molecules differ in their rudimentary structure in such a way that they behave like images and mirror images³.

It is shown, one form of these of molecules rotates the plane of polarization of the light in the mathematically negative direction (right-hand rotation), the other kind rotates the polarization plane counter clockwise (or mathematically positive direction). One then speaks of the (+)-form/D-form or the (-)-form/L-form of the molecule.

In this experiment, treacle will be investigated, since all varieties of sugars belong to the optically active substances. In the experiment the monosaccharides (single sugar) glucose and fructose are examined, as well as sucrose as representatives of the class of the disaccharides⁴ (Polysaccharides). If a solution is a 1:1 mixture of the D- and L-forms of a molecule, the optical activity will be exactly cancelled. This is exactly the case with all other substances, which are called non-optically active. In fact, non-optically substances polarize light as well, but the rotation of polarization plane is cancelled at another molecule, resulting on average in light in initial state. For these substances, however, there exist no enantiomers and thus, there can never be a case in which a rotation takes place.

For enantiomers such as L-fructose and D-fructose, this is different. In this case, there are no molecules cancelling the rotation of the polarization plane.

The angle α , at which the plane of polarization is rotated, depends on the mass concentration C_m of the sugar solution, on the length l of the path passing through the solution, and a material-specific quantity, which is called specific rotation α_s . The following applies:

$$\alpha = \alpha_s \cdot l \cdot C_m \quad (1.8)$$

If the specific rotation of e.g. D-fructose is known, one can determine the concentration of an arbitrary solution - this is called Saccharimetry. This statement may be generalized to all other optically active substances.

Why this rotation takes place at all is a very difficult question, to which only quantum mechanics can give an answer to. This would go beyond the scope of this manual. Therefore an exact explanation of this phenomenon must be omitted here.

1.3 Experiment

The setup of the experiment consists of a halogen lamp, which serves as a light source, a cuvette with a sugar solution, located between a polarizer and an analyzer. A lens and an screen allow the optical image to be observed. The whole apparatus is located on an optical track.

³It is often said, they behave like the left hand to the right hand.

⁴For example, also cellulose, maltose, and lactose are part of this class. All of which are of great importance in biochemistry.

The experiment has to be set up and the optical image has to be adjusted. Position the halogen lamp at the end of the optical track and connect it to the power supply. Now place the polarizer and the analyzer on the rail. Between these two devices, you must now position the cuvette. The scale of both the analyzer and the polarizer have to point at the screen and they have to be set to 90° . Subsequently, position the lens and the observation screen. The focal length of the lens is noted on the holder of the lens, it must be positioned accordingly (think about what actually has to be in the focal point of the lens).

Then, the lamp, the lens and the screen have to be aligned, with help of the adjusting rod, in order to get an uniform illumination of the field of view at the screen.

Before you start filling the cuvette with treacle, show your setup and your adjustments to your assistant.

Measurements are done as follows:

- Set the polarizer to 90°
- Set the analyzer to 0°
- Prepare your solution in a beaker. Make sure that all sugar is dissolved.
- Fill the cuvette with the solution.
- Now turn the analyzer and measure the entire light field.
- Note the setting of the analyzer at which you observe the minimum intensity on the screen and which colour filter was used.

Used sugar solutions can be washed down the drain. Clean the cuvette thoroughly with distilled water, before using it for the next measurement.

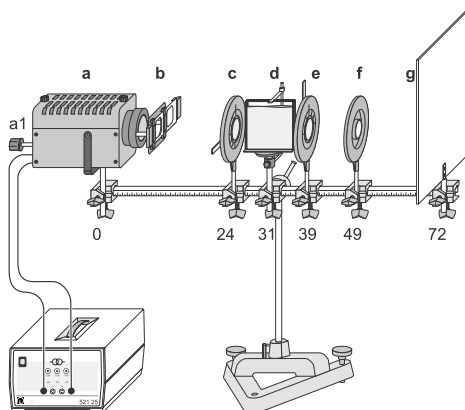


Figure 1.2: The experimental setup consists of a halogen lamp (a), colour filters (b), a Polarizer (c), a cuvette with treacle (d), an Analyzer (e), a lens (f) and a screen (g).

1.3.1 Tip

We highly advice (but not force) to copy (without the incorrect example line) and complete the following chart in order to write down the measurements of your experiments:

Compound	Light Filter	Mass concentration C_m in [g/cm ³]	Mass m used to prepare the solution in [g]	Volume V used to prepare the solution in [L]	Rotation angle [°]
Sucrose	Yellow	30	525 ± 20	25 ± 15	90 ± 15
...	...				

1.3.2 Equipment

Item	Quantity
Halogen lamp	1
Power supply	1
Analyser	1
Polarizer	1
Cuvette	1
Colour filter (red, green, blue)	3
Lens, f=100 mm	1
Observation Screen	1
optical rail	1
Muffle	6
Stand	1

1.3.3 Experimental Setup and Adjustment

- Measure the length of the cuvette and determine the corresponding measurement uncertainty.
- Prepare a solution of 0.20 g cm⁻³ of sucrose. Note the mass of sucrose and the volume of water that you use and estimate for each the error on these values.
- Irradiate this solution with white light, without any colour filter. Look at what happened on the screen when you slowly turn the analyzer all the way (from 0° to 360°).
What is your observation?
What can you conclude? Pay special attention to the resulting colour gradient close to the extinction.
Call your assistant to discuss about your observations and the resulting conclusion.
- Using now the green light filter, measure the rotation angle of this solution. Evaluate the error on your measurement.
- Repeat the steps b) and d) with three other sucrose mass concentrations: 0.30 g cm⁻³; 0.40 g cm⁻³ and 0.50 g cm⁻³.
- Prepare a solution of 0.25 g cm⁻³ of glucose. Again, note the mass of glucose and the volume of water that you use and estimate for each the error on these values.
- Use all three colour filters to determine the rotation angles of the glucose in each case.

- h. Repeat the two previous steps with a solution of fructose.
- i. Additionally, you will get a sucrose solution with an unknown concentration. Measure this solution as well.

1.4 Tasks for Evaluation

- Describe in detail your observation at the screen, when irradiated with white light.
- Create a graph of $\alpha(C_m)$ from your measurements of sucrose solutions with different concentrations including on both axis the errors (calculated via error propagation formula for the concentration).
- Make a linear fit to this plot and determine the specific rotation from its slope. Calculated the error on this value applying propagation formula on formula (1.8).

NOTE: One should notice that we are not supposed to use linear regression when the errors on x axis are not negligible. We make the assumption in this experiment that we can use this method nevertheless, avoiding a more accurate but time-consuming analysis.

- Use the measured α_s values to determine the molar concentration of the unknown sucrose solution⁵. Given the volume of the initial solution was $V = 1.0 \text{ L}$, calculate the amount of sucrose dissolved in it in grams.
- Create a graph of $\alpha(\lambda)$ from your fructose and glucose measurements⁶. Determine the proportionality of your plot. Is it linear or proportional to a parabola? Try to give a physical interpretation of your results.
- Sucrose is a molecule made by bounding one molecule of glucose with one of fructose. Is this fact reflected in your data in any way, or is this experiment not suitable for making such a statement?
- Do a little research to find out the known value of α_s for sucrose. How different is the value you found from literature value? Is the literature value within the error of your experimental result? If it is not, try to address possible reasons for it.

NOTE: when you compare two values, pay attention that both values have the same unit!

Literature

- Demtröder Band 2 - *Elektrizität und Optik*, 6. Auflage: Abschnitt 7.4 und 8.5
- E. Hecht - *Optik*, 5. Auflage: Abschnitt 8.1 bis 8.6, sowie 8.10

⁵The molar mass of fructose is equal to the molar mass of glucose $M = 180.16 \text{ g/mol}$. The molar mass of sucrose is $M_{suc} = 342.3 \text{ g/mol}$.

⁶The wavelengths of the colour filters are: $\lambda_{red} = 720 \text{ nm}$, $\lambda_{green} = 530 \text{ nm}$, $\lambda_{blue} = 450 \text{ nm}$