# IO1

**Modul Optics**

# Geometrical optics and lens equations

In this experiment, the laws of geometical optics are investigated. Gauss's method is used to verify the lens equation for two lenses with different focal length and to study lens doublets. Additionally, optical aberration is examined in more detail.

# Versuch IO1 - Geometrical optics and lens equations

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## 1.1 Preliminary Questions

- What is meant by the term geometrical optics? When can geometrical optics be applied?
- What does Fermat's principle say?
- What is meant by the terms reflection and refraction?
- What are the first and the second statement of the intercept theorem (also known as Thales's theorem)?
- Make a sketch of the beam path through a concave lens as well as through a convex lens.
- What is meant by the term thin lens? Explain!
- What is the so-called thin lens equation and when is it valid?
- Why is an optical image not automatically sharp at the focal point?
- What imaging errors occur with optical lenses?

## 1.2 Theory

### 1.2.1 Introduction

To describe the propagation of light, it is in principle necessary to consider its wave nature. However, the typical wavelengths of light are so small that wave effects are generally not observed. In that case, the propagation of light can be described by FERMAT's principle. For this, we consider the optical density $n(\vec{r})$ (refractive index) along a path traversed by light. FERMAT's principle states that between two points A and B, light takes the path for which the integral

$$\int_A^B n(\vec{r})dr \qquad (1.1)$$

becomes *extremal*. Extremal means that the integral is either maximal or minimal along the respective path, or stationary, meaning it does not change for infinitesimally neighboring paths (analogous to the extremal points of a function, which can be a maximum, minimum, or a saddle point). In practice, an equivalent formulation is used, based on three laws.

1. Straight-line propagation of light in a *homogeneous* and *isotropic*[1] medium.
2. Law of Reflection: When light is reflected, the angle of incidence is equal to the angle of reflection.
3. Law of Refraction: The angles $\alpha$ of the incident ray and $\beta$ of the refracted ray satisfy the relationship

   $$n_1 \cdot \sin(\alpha) = n_2 \cdot \sin(\beta) \qquad (1.2)$$

   where $n_1$ represents the optical density in the medium of the incident ray and $n_2$ in the medium of the refracted ray.

[1]Homogeneous means that the corresponding properties within the medium do not vary, i.e., they are independent of location, while isotropic means direction-independent.

In this context, all angles are measured with respect to the normal of the surface between the media. These are the axioms of *geometrical* optics. Geometrical because light propagates here in the form of rays, allowing for a simple geometrical treatment. Geometrical optics is valid whenever all dimensions of the path through which the light wave travels are much larger than the wavelength, and when there are no superpositions of waves of the same wavelength. The Fresnel zone plates are an impressive example of a phenomena in *wave optics*, where the above-mentioned conditions are no longer fulfilled due to the microwaves used and their wavelength in the centimeter range. An everyday example that cannot be explained by geometrical optics is the color patterns appearing on an oil film, as the thickness of the oil layer is of the same order of magnitude as the wavelength of light. In the further course of this guide, unless otherwise stated, we assume that the above-mentioned conditions are met, and we are within the validity range of geometrical optics.

### 1.2.2 Imaging systems & lenses

In an optically inhomogeneous medium or in a system of media with different optical densities, light rays will no longer propagate in straight lines. In an *ideal imaging system*, rays emanating from a common point or converging to a common point will also intersect at a common point after passing through the system. This allows points of an object to be mapped onto corresponding image points. In every *real imaging system*, there are various deviations from this ideal behavior. Imaging systems can be easily realized using *spherical lenses* or mirrors. Spherical lenses have two spherical surfaces, which can be either convex or concave. The line connecting the centers of the spheres is referred to as the *optical axis*. Depending on whether the lens is thicker or thinner in the middle compared to the edge, we speak of *converging lenses* or *convex lenses*, which deflect rays towards the optical axis, or *diverging lenses* or *concave lenses*, which deflect rays away from the optical axis. A bundle of rays parallel to the optical axis, upon passing through a converging lens, converges at a point behind the lens known as the *focal point*. A diverging lens, on the other hand, will disperse a parallel bundle of rays as if they were coming from a focal point located in front of the lens.

### 1.2.3 Thin lenses

The imaging properties of lenses are based on the refraction of light rays passing through the front and back surfaces of the lens.
We now want to trace the path of light rays through a converging lens (see Fig. 1.1). A ray emanating from point A at an angle $\alpha_1$ to the optical axis strikes the lens at point H1 at a height of h1, undergoes refraction within the lens with the refractive index $n_2 = n$, exits the lens at point H2 at a height of h2, and finally strikes point B again on the optical axis at an angle $\alpha_2$. From the diagram, it can be seen that the angle of incidence at the first refraction is $\alpha_1 + \beta_1$, and with the refractive index $n_1 = 1$ for the first medium (air), the refraction law is given by

$$\sin(\alpha_1 + \beta_1) = n \cdot \sin(\gamma_1). \tag{1.3}$$

Similarly, upon exiting the lens, the refraction law is

$$n \cdot \sin(\gamma_2) = \sin(\alpha_2 + \beta_2). \tag{1.4}$$

Now, considering that $\gamma_2 = \beta_1 + \beta_2 - \gamma_1$, we obtain for the second refraction:

$$\sin(\alpha_2 + \beta_2) = n \cdot \sin(\beta_1 + \beta_2 - \gamma_1). \tag{1.5}$$

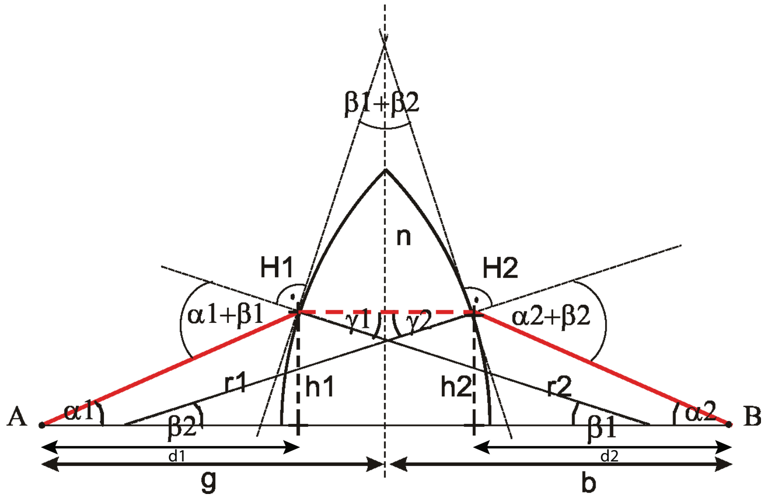


Figure 1.1: Diagram of the ray path in a converging lens. (In this drawing r1=r2 applies, leading to the symmetry of the image.)

For nearly axial (paraxial) light rays, or for lenses with small diameters, the angles involved are small, and we can replace the sines with their arguments. Adding the approximated equations 1.3 and 1.5 for both refractions yields

$$\alpha_1 + \alpha_2 = (n-1)(\beta_1 + \beta_2). \tag{1.6}$$

Approximating the small angles in this equation with $\alpha_i = h_i/d_i$ and $\beta_i = h_i/r_i$, we obtain

$$\frac{h_1}{d_1} + \frac{h_2}{d_2} = (n-1)\left(\frac{h_2}{r_1} + \frac{h_1}{r_2}\right). \tag{1.7}$$

Since we are considering a thin lens, $h_1 \approx h_2$, and we get

$$\frac{1}{d_1} + \frac{1}{d_2} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right). \tag{1.8}$$

For the special case where the rays come from an infinitely distant point, i.e., it is a parallel bundle of rays, we know that the refracted rays will intersect behind the lens at the focal point. Let $f$ be the *focal length* of the lens, i.e., the distance from the lens to the focal point. In this case, $d_1 \to \infty$ and $d_2 \to f$, and equation 1.8 becomes

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right). \tag{1.9}$$

The focal length of a lens is therefore smaller, the smaller the curvature radii $r_i$ of its bounding surfaces are. The curvature radii of concave surfaces are used with a negative sign. From equations 1.8 and 1.9, and with the *object distance* $g \approx d_1$ and the *image distance* $b \approx d_2$, being the distances from the object and the image to the lens, respectively, we obtain the well-known lens equation[2]:

$$\frac{1}{g} + \frac{1}{b} = \frac{1}{f}. \tag{1.10}$$

Thus, the rays from an object at a distance $g$ will intersect at a distance $b$ behind the lens, allowing the object to be projected onto a surface at a distance $b$. This, however, applies only

[2]For consistency we use the same notation as in the German lab manual.

to converging lenses. For diverging lenses, the focal length is negative, and the divergent rays intersect *before* the lens. Therefore, the image of a single diverging lens cannot be projected onto a surface.
Analogous considerations show that for a system of two lenses with focal lengths $f_i$ separated by a distance $d$, we have:

$$\frac{1}{f_{\text{doublet}}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2} \approx \frac{1}{f_1} + \frac{1}{f_2} \quad \text{for} \quad d \ll f_1, f_2. \tag{1.11}$$

In literature, instead of the focal length $f$, you often find its reciprocal, the *optical power* $D = 1/f$. The unit of optical power is $[D]$ = dioptre (dpt) = $\mathrm{m}^{-1}$.
When the focal length $f$ of the lens is known, the position and size of the image of a given object can be found through a simple construction, see Fig. 1.2. In the construction, the lens is replaced by its *principal plane* $M$. The space on the object side enclosed by the lens is also called the *object space*, and on the image side, the *image space*.

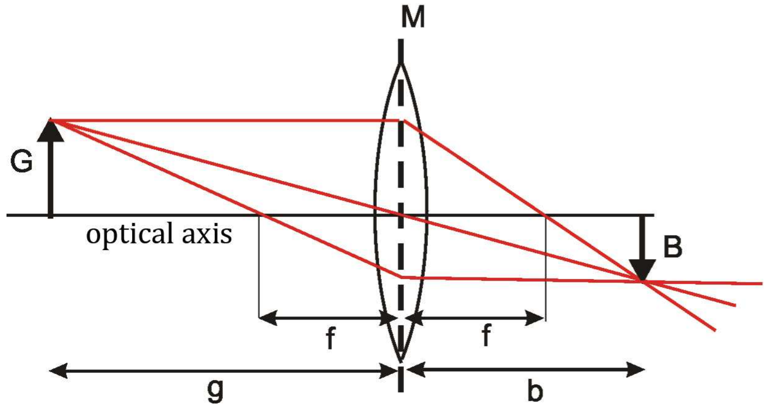


Figure 1.2: Construction of the image $B$ of an object $G$ for a known focal length $f$ of the lens.

For a thin lens (given by its principal plane and its focal length) parallel rays from the object space pass through the focal point of the image space, and vice versa. Rays passing through the center of the lens are not deflected and travel in straight lines. From Figure 1.2, it can be seen[3] that for the size ratio between object and image, i.e., for the *magnification* $M$, we have:

$$M := \frac{B}{G} = \frac{b}{g}. \tag{1.12}$$

Depending on the distance between the object and the lens, it produces an enlarged or reduced image. The images are termed *real* or *virtual*, depending on whether they are formed in the image space or the object space. They are called virtual because such images cannot be projected onto a surface. Virtual images can only be produced with diverging lenses, and conversely, real images only with converging lenses.

### 1.2.4 Lens aberrations

In spherical lenses with larger diameters and stronger curvature, a zone error becomes noticeable, the *spherical aberration*, where due to the approximations in equations 1.7 and 1.10, deviations from equation 1.10 occur for off-axis rays. Consequently, rays entering the lens

[3] Applying the intercept theorem (Thales's theorem) to the rays passing through the center of the lens.

further from the center are refracted more strongly than those closer to the lens' optical axis. Thus, there is not a single focal point; rather, the rays are focused on different focal points.
Another important deviation from the ideal behavior according to equation 1.10 is *chromatic aberration*, which occurs due to the different refractive indices of the lens material for different wavelengths. For example, the focal length for blue light is shorter than for red light.
In addition to these two main effects, there are other lens aberrations (astigmatism, coma, etc.). Lens aberrations can be partially corrected through clever lens combinations (different types of glass, curvatures, etc.).

## 1.3 Experiment

The setup of our experiment consists of a rail, called an optical bench, on which various optical elements are mounted as shown in Fig. 1.3. By adjusting the positions of these elements relative to each other, the desired experimental configurations can be set up. You will find the experiment in a disassembled state, and it will be your task to setup the correct ray paths according to the theory outlined above. The exact alignment and adjustment of all optical devices can be time-consuming. Good measurement data can only be achieved through skill and diligence in adjustment - so give it your best effort! Pay attention to the precise path of

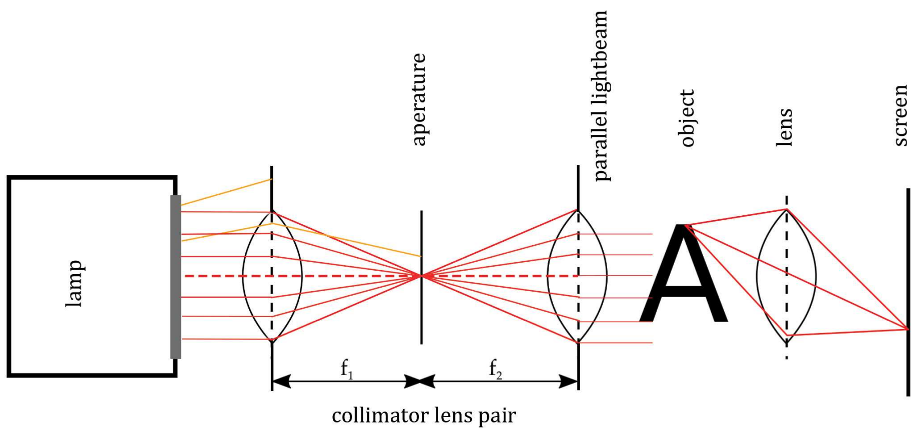


Figure 1.3: The experimental setup features a lamp, whose slightly divergent light is collimated using two converging lenses and an aperture. The now parallel beam of light illuminates an object, which is projected onto a screen using one or two imaging lenses.

the rays and try to realize it as accurately as possible during the experiment. Handling by many students may result in some of the lenses or other devices being dirty. If necessary, these should be cleaned. Isopropanol is available in the laboratory rooms for cleaning lenses.

### 1.3.1 Equipment

| Object | Quantity |
|---|---|
| Lamp, mounted on optical bench | 1 |
| Lens | 8 |
| Slide | 3 |
| Slide holder | 3 |
| Set of color filters | 1 |
| Meter stick | 1 |
| Screen | 1 |

### 1.3.2 Experimental procedure

A) *Experimental setup*. How can a parallel beam of light be generated? Set up the experiment according to Fig. 1.3 and discuss it with the assisting person.

B) *Lens equation*. Verify the lens equation (1.10) using the so-called Gaussian method for two converging lenses with different focal lengths. Place a slide (object) near the collimator lens pair. Move the screen until the image is in focus and measure the image distance $b$ and the object distance $g$. Then, move the lens, adjust the screen, and measure the image and object distances again. Repeat this process until you have at least 15 pairs of values. For the evaluation, plot the reciprocals of the object distance $g$ on the x-axis and the reciprocals of the image distance $b$ on the y-axis, which will result in a straight line from which the focal length can be extracted. Before the measurement, consider how to obtain a meaningful distribution of value pairs. Then, perform the same experiment with another converging lens.

C) *Lens doublets*. Show that for lens doublets $1/f_1 + 1/f_2 = 1/f_{\text{doublet}}$ holds.

a) Determine the focal length of a system consisting of two thin lenses mounted at a distance $d$ from each other on the optical bench. Use the same method discussed above. Here, too, at least 15 pairs of values are to be recorded. Use converging lenses for this purpose.

b) Explain the difference between real and virtual image. Then, determine the focal length of a diverging lens. Consider how the virtual image can be projected onto the screen and sketch it accordingly.

D) *Observation of spherical and chromatic aberration*. Various slides and objects, as well as several color filters, are available for observing the different types of aberrations.

a) Consider how the spherical and chromatic aberrations can be observed using the available materials. Perform these experiments and document your observations.

b) [Optional - for advanced students.] Using the Gaussian method and a color filter, determine the focal length of a lens that you have already measured.

## 1.4 Evaluation

- Prepare a detailed report describing all conducted experiments, including a sketch illustrating the respective ray path of the setup. Additionally, all calculations including error estimations or error calculations are to be presented.

- For the evaluation of measurements using the Gaussian method, plot the reciprocals of the image distance $b$ (y-axis) against the reciprocals of the object distance $g$ (x-axis). This will result in a straight line. What is the slope of this line, and where does it intersect the y-axis? What does the symmetry of the obtained function express physically?
- Sketch the ray paths for spherical aberration and chromatic aberration. Describe precisely which objects and methods you used to observe these two aberrations and explain your observations.

## Literature


- Demtröder Band 2 - *Elektrizität und Optik*, 6. Auflage: Abschnitt 9.1 bis 9.5
- E. Hecht - *Optik*, 3. Auflage: Kapitel 5