IM6

Modul Mechanics

Centrifugal force

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Versuch IM6 - Centrifugal force

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1.1 Preliminary Questions

- What is a central force?
- What is the difference between the centripetal force and the central force?
- What is the centrifugal force?
- What is the difference between centrifugal force and centripetal force?

1.2 Theory

1.2.1 The Equal Circular Motion

If a mass point moves on a circular path with radius r around the center Z with a constant velocity v, the *path velocity* v denotes the arc length of the circle traversed in one second. Furthermore, the angle traversed by a ray from the center Z to the mass point in a given time ü divided by this time is given by the *angular velocity* ω. The angle is to be given here in radians. The relation between path velocity and angular velocity is thus given by

$$v = \omega r \tag{1.1}$$

The time needed for the mass point to make one revolution is then calculated as follows

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} \tag{1.2}$$

A rotating rigid body, e.g. a wheel, has the same angular velocity at all points, but because of (1.1) the orbital velocity increases outward.

Since the direction of the velocity changes, there must be an acceleration. We now want to determine this acceleration of the uniform circular motion. According to the general method of kinematics, this acceleration can be found by forming the velocity difference füfor two positions A and B of the mass point, which are sufficiently close to each other, or the corresponding time points t_1 and t_2 (see Fig. 1.1). The circle sector ZAB can now be approximated with arbitrary accuracy by a triangle. This triangle is similar to the triangle BCD (from the two velocity vectors $v_1(t)$ and $v_2(t)$) and the velocity difference Δv. Both triangles are isosceles and both have the same angle at the apex. If Δr is the length of the arc, which in the boundary case merges into the triangle side ü, then corresponding sides of both triangles have the same ratio:

$$\frac{AB}{r} = \frac{\Delta r}{r} = \frac{|\Delta v|}{|v|} = \frac{|\Delta v|}{v}$$

If we divide this equation by the time difference $\Delta t = t_2 - t_1$, which is needed to cover the distance AB or to change the velocity Δv, we get:

$$\frac{\Delta r/\Delta t}{r} = \frac{v}{r} = \frac{|\Delta v|/\Delta t}{v} = \frac{a}{v}$$

If we now insert the acceleration a for $\Delta r/\Delta t$ in the limiting case v and for $|\Delta v|/\Delta t$, we obtain the expression for the acceleration of the uniform circular motion, the *centripetal acceleration*:

$$a = \frac{v^2}{r} = \omega^2 r \tag{1.3}$$

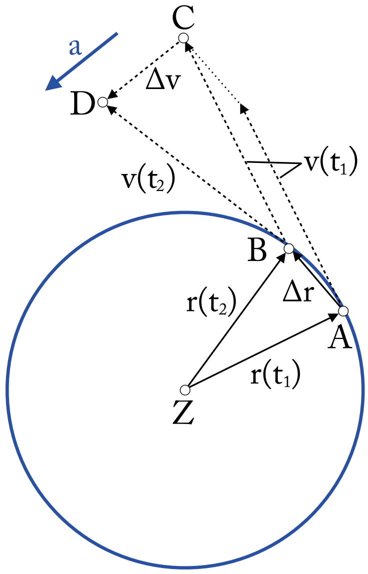

Figure 1.1: Kinematics of uniform circular motion, with the center (Z) of the circle. The movement on the circle leads from location A at time t_1 to location B at time t_2.

The magnitude of this acceleration is therefore constant and its direction is given by the construction in Fig. 1.1 as always directed to the center. Note that Δv and a are actually to be applied at location A and B. In order for or when a body of mass m performs a uniform circular motion, a force, the *centripetal force*, of magnitude

$$F = \frac{mv^2}{r} = m\omega^2 r \tag{1.4}$$

which always points to a fixed point, the center. In our case, the metal body is moved outward by the rotation due to its tricity and is held on its circular path by the wire attached to the torsion wire.

1.3 Experiment

In the present experiment, the Schürholz radial force device is used to determine the radial force, *the centripetal force*, as a function of mass, angular velocity and distance of the rotating bodies from the axis. When the device is rotated, the twist of the torsion thread is proportional to the radial force and is made visible by a light pointer. To derive the radial force law, one changes the mass, distance and angular velocity of the rotating metal body.

1.3.1 Equipment

Component	Number
Radial force device	1
Experiment motor	1
Control and regulation unit for the experimental motor	1
Lamp	1
Light pointer slide	1
Tripod rod 25 cm	1
Sleeve	1
Tripod base	1
Transformer	1
Height scale	1
Base	1
Precision force gauge 1.0 N	1
Hand stopwatch	1

1.3.2 Experimental Procedure and Adjustment

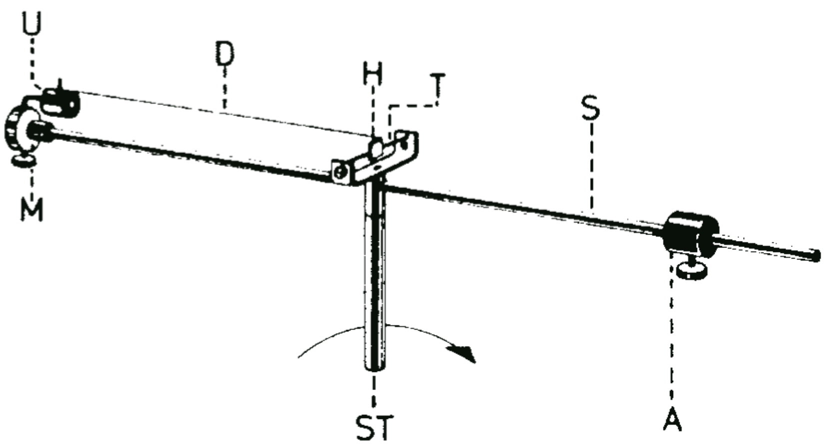

Figure 1.2: Radial force device according to Schürholz. The radial force device is mounted on the experimental motor with rotation axis (ST). The metal body (U) is placed on the pin of the sleeve (M) and connected to the concave mirror (H) by a wire (D). Opposite the metal body, the balancing body (A) is mounted.

- The radial force device is fixed in the clamp of the experiment motor.
- The metal body (U) to be used (Fig. 1.2) is placed on the pin of the sleeve (M) and hung on the hook of the mirror (H) with one of the two wires (D). The sleeve is pushed either to the inner or to the outer stop, so that either the short or the long wire fits exactly.
- The balancing body (A) is clamped at approximately the same distance from the axis.
- To set up the light pointer reading, place the lamp with light pointer slide in the condenser on the stand base with a socket and a stand rod. The stand base is placed about 30cm from the concave mirror (H) and the lamp is adjusted so that the condenser creates a magnified image of the lamp filament on the concave mirror.

- The light pointer slide in the condenser is imaged through the concave mirror onto the height scale at a distance of at least 2m. To focus this image, the lamp is shifted.
- When the axis is rotated (approx. 0.5 to 1 revolution/second), the image is periodically visible on the screen for a short time, but it is sufficient to read the deflection with the help of the scale.
- To calibrate the light pointer deflection in Newtons, hang the force gauge in place of the metal body (U) in the torsion wire and read related values of force and light pointer deflection.

1.3.3 General

For all measurements, record the errors of the measurements and enter them together with the measured values in the log. The number of decimal places of the error corresponds to those of the measured value. E.g.: $H = (24.5 \pm 0.1)$ cm.
Determine the error of the measured value for all results using the error propagation. Example:

$$F = m \cdot r \cdot w^2$$
$$\text{mit } m = (25.20 \pm 0.01) \cdot 10^{-3}\text{ kg}, r = (12.0 \pm 0.1) \cdot 10^{-2}\text{ m}, w = (15.65 \pm 0.25)\text{s}^{-1}$$

In this case, the error is calculated via the relative errors. This simplification of the general error propagation is equivalent to the general form (with partial derivatives), provided that the formula does not contain any sums or differences:

$$\frac{\Delta F}{F} = \sqrt{\left(\frac{\Delta m}{m}\right)^2 + \left(\frac{\Delta r}{r}\right)^2 + 4 \cdot \left(\frac{\Delta w}{w}\right)^2}$$

The number of significant digits of the result is given by the number of significant digits of the worst measured value, which is included in the calculation. For the above example:

$$F \widehat{=} F \pm \Delta F = (0.74 \pm 0.03)\text{ N}$$

1.3.4 Measurements

- Using the precision force meter, measure the light pointer deflection as a function of the force pulling on the mirror. Note the light pointer deflection in steps of 0.1N and plot it graphically against the force. Determine the calibration function using a linear fit of the form:
$$H(F) = a + b \cdot F$$
with the deflection of the light beam without force effect a and the proportionality constant b.
Via the inverse function of the calibration function can be converted to the applied force in later height measurements.
- In the following, the deflection of the light beam as a function of the angular velocity is determined for each combination of two different masses and two different wires. For this purpose, the height is determined for each mass/wire combination for 8 different angular velocities. The angular velocities are to be selected in such a way that the entire range covered by the calibration curve is nearly utilized. The angular velocity is determined by measuring the duration of 20 revolutions using a stopwatch.

1.4 Evaluation

In all graphs, a legend must explain the plot. In addition, the errors of all measuring points must be shown by error ba

- The data of the measurement (force versus angular velocity) are plotted graphically and fitted with a parabola of the form:

  $$F = a \cdot w^2$$

  The fit parameter a contains the information about the mass and the wire length. It is necessary to check how the fit parameter scales with the change of the mass (wire length). Does the ratio agree with the expected value within the error?

- The data of the measurement (force versus angular velocity) are plotted double logarithmically and fitted with a linear function. The slope of the straight line is the exponent of the dependence of the force on angular velocity. Does the slope agree with the literature value within the error?

Literature

- D. Meschede, "*Gerthsen Physik*", Springer Verlag