IM2

Modul Mechanics

Air Table

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Versuch IM2 - Air Table

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1.1 Preliminary Questions

- What are the fundamental conservation laws of physics? To which quantities do they apply under what conditions?
- What is an inertial system?
- What is a conservative force?
- Deduce the conservation of momentum from NEWTON'S LAWS OF MOTION.
- What is a centre-of-momentum frame and how do you calculate is centre of mass?

1.2 Theory

1.2.1 Newton's Laws of Motion

The principles of motion, noted by ISAAC NEWTON in 1687, called *Newton's laws of motion* build the foundation of classical mechanics. Although, within scope of modern physical theories like quantum mechanics or relativity theories, they are not unconditionally valid, but can still give acceptable prediction within a vast scope of applications.

Newton's First Law - *Lex Prima*

Newton's law first law, also called PRINCIPLE OF INERTIA, describes the motion of physical bodies within an inertial reference frame, in absence of external forces. It declares, that a body in a state of uniform translation of at rest does not change its condition, as long as there are no additional forces applied.
Within those conditions, the body's velocity, its magnitude and its direction are constant. To change its state of motion, a external force (e.g. a gravitational force) has to be applied.

Newton's Second law - *Lex Secunda*

Newton's law second law provides the foundation for most equations of motion in classical mechanics. It states that the rate of change of momentum of a body, is directly proportional to the force applied and this change in momentum takes place in the direction of the applied force. In mathematical terms this correlation is described as

$$\dot{\vec{v}} \propto \vec{F}$$

and was stated in Newton's original work in its universal formulation

$$\vec{F} = \dot{\vec{p}} \, .$$

Since 1750, the following form

$$\vec{F} = m\vec{a} \tag{1.1}$$

stated by Leonhard Euler is known as *Fundamental equation of mechanics*, where $\vec{a}$ describes a change of velocity in time, also known as *acceleration*.

Newton's Third Law - *Lex Tertia*

Newton's third law, the INTERACTION PRINCIPLE, states that to every action there is always opposed an equal reaction. A body 1 that forces an action upon a body 2, experiences the same force, but in opposite direction:

$$\vec{F}_{1\to 2} = -\vec{F}_{2\to 1}$$

Hence, in a closed system the sum of all forces are equal to zero:

$$\sum_i \vec{F}_i = 0 \tag{1.2}$$

Newton's third law is also known as the principle of *actio and reactio*. Although the requirement of a long range implies, that it loses its general validity within the theories of time dependent electrodynamics and in special relativity and applies only under certain conditions.

Newton's Fourth Law - *Lex Quarta*

Newton's fourth law was only accounted for as an addition in his original work and became *lex quarta* later on. It describes the principle of uninterrupted superposition and states the net response of several forces on one point or rigid body is the sum of all individual forces:

$$\vec{F}_{res} = \sum_i \vec{F}_i \tag{1.3}$$

Conservation of Momentum

The conservation of momentum is one physics fundamental laws and states, that the total momentum in a closed system is preserved or constant. This law in independent from the law of conversation of energy ,and it is also valid for theories like classical mechanics, quantum mechanics and special relativity. For collision processes this indicates, that total momentum before and after a collision must be equal. This is valid for elastic (when kinetic energy is preserved during the collision) as well as for inelastic collisions (when kinetic energy is lost during the collision).
The conservation of energy is a direct implication from Newton's second and third law. Since the force acting upon a body is equal to the change in time of the momentum (Newton's second law):

$$\vec{F} = \dot{\vec{p}}$$

and because there is an equal, opposing force to every force in this system (as long as there is no external force[1]), the sum of all forces is zero. Since this is true for all forces, this also implies that the sum of all vectors, acting in this system is equal to zero. And therefore the sum of all changes in time of all momentums:

$$\vec{F} = \sum_i \vec{F}_i = \sum_i \dot{\vec{p}}_i = \dot{\vec{p}} = 0. \tag{1.4}$$

Since the time derivative of the momentum vanishes, the momentum itself is constant and therefore the centre of mass moves with constant velocity. This leads to the conclusion, that the centre of mass of a system moves with constant velocity and direction, as long as external forces are absent.

[1]For the conservation of momentum, the requirement of no external forces is not complete necessary. It is sufficient to claim, that the sum of all external forces is equal to zero $\sum_i \vec{F}_i^{\text{ext}} = \vec{F}^{\text{ext}} = 0$. Therefore, the individual forces do not have to vanish, but only the sum of all external forces.

Conservation of Energy

In Newton's mechanics, the total energy E of a system consists the sum of kinetic energy T and potential energy V. And in case of bodies moving in a conservative field, this total energy is conserved. Whereas the vector of the force $\vec{F}$ is equal to the negative gradient of the potential energy:

$$\vec{F} = -\vec{\nabla} V$$

A particle moving through a conservative field, in time t, on an arbitrary path $x(t)$, will always do the same work, defined by the difference in potential energy of its start and end point.

With Newton's first law (Eq.: 1.2.1) we can state the following:

$$m\ddot{\vec{x}} = \vec{F} = -\vec{\nabla} V.$$

Multiplying both sides with $\dot{\vec{x}}$ will give us:

$$\begin{aligned} m\ddot{\vec{x}}\dot{\vec{x}} &= -(\vec{\nabla} V)\dot{\vec{x}} \\ &= -\sum_{i=1}^{3} \frac{\partial V}{\partial x_i}\frac{dx_i}{dt} \\ &= -\frac{dV}{dt} \end{aligned}$$

Integration of this term over time leads to the work needed along an arbitrary, continuously differentiable path with the potential energy V_1 at start point and V_2 at end point:

$$\begin{aligned} \int_{t_1}^{t_2} m\ddot{\vec{x}}\dot{\vec{x}}dt &= -\int_{V_1}^{V_2} dV \\ T_2 - T_1 &= -V_2 + V_1 \\ T_1 + V_1 &= T_2 + V_2 \end{aligned}$$

Thus, the sum of potential and kinetic energy stays the same before and after the body was moving, hence the total energy is *conserved*.

Centre-of-Mass Theorem

The centre-of-mass theorem states, that the centre of mass of a multi-body system acts like a point-mass, which has the combined mass of all bodies in this system and is affected by the sum of all force-vectors acting on all the individual bodies.
Hence the centre of mass is moving unaffected of all the inner forces acting between the individual bodies. If all external force-vectors add up to zero, the centre of mass is moving linear, free of forces and without any change in velocity (Newton's first law).

1.2.2 Elastic collisions

In an ideal elastic collision no energy gets lost, as it is the case in inelastic collisions. In the elastic case energy- and momentum-conservation hold, meaning the sum of the kinetic energies and the momenta before and after the collision are equivalent.

$$\frac{1}{2} \cdot m_1 \cdot v_1^2 + \frac{1}{2} \cdot m_2 \cdot v_2^2 = \frac{1}{2} \cdot m_1 \cdot v_1'^2 + \frac{1}{2} \cdot m_2 \cdot v_2'^2 \tag{1.5}$$

$$m_1 \cdot v_1 + m_2 \cdot v_2 = m_1 \cdot v_1' + m_2 \cdot v_2' \tag{1.6}$$

These two equations can be solved for the velocities v'_1 and v'_2 after the collision:

$$v'_1 = \frac{m_1 \cdot v_1 + m_2 \cdot (2v_2 - v_1)}{m_1 + m_2} \tag{1.7}$$

$$v'_2 = \frac{m_2 \cdot v_2 + m_1 \cdot (2v_1 - v_2)}{m_1 + m_2} \tag{1.8}$$

In part (b) of the tasks the different parameters like the masses and the velocities should be chosen appropriately to get the desired results.

1.2.3 Inelastic collisions

In an inelastic collision the two original objects are moving as one mass after the collision, they stick together. The equations from the elastic collision therefore change slightly:

$$\frac{1}{2} \cdot m_1 \cdot v_1^2 + \frac{1}{2} \cdot m_2 \cdot v_2^2 = \frac{1}{2} \cdot (m_1 + m_2) \cdot v'^2 + \Delta E \tag{1.9}$$

$$m_1 \cdot v_1 + m_2 \cdot v_2 = (m_1 + m_2) \cdot v' \tag{1.10}$$

$$v'_1 = \frac{m_1 \cdot v_1 + m_2 \cdot v_2}{m_1 + m_2} \tag{1.11}$$

1.3 Experiment

1.3.1 Accessories

- (1) Stable base of the three-point support.
- (2) Adjusting screws for additional feet (to stabilize the level adjustment obtained with the three-point support (1)/(3))
- (3) Levelling screws for the three-point support
- (4) Recess for metallized recording paper
- (5) Power supply for puck fan and recording electrodes
- (5.1) Frequency selector (10 Hz/50 Hz) for applying spike pulses to recording electrodes (12.5) and/or (18).
- (5.2) Power switch with mains indicator lamp.
- (5.3) Holder with primary fuse.
- (5.4) Socket for power-supply arm (10).
- (6) Clamping strip for metallized recording paper and for providing electrical contact (recording circuit).
- (7) 4-mm-sockets, internally connected with clamping strip (6) and power supply (recording circuit).
- (8) Key switch for switching the registration pulses on and off.
- (9) Roll of metallized recording paper, 20m length, 45cm width (consumable material).

(10) Power supply arm, pluggable into socket (5.4); with two sockets connected in parallel to connect the supply lines (11) for two pucks (12).

(11) Power lead (2x), approx. 85 cm long, for voltage supply from the power supply (5) to the pucks (12).

(12) Puck (2x) with fan for producing the air cushion and with centre electrode lightly dragging on the recording paper. (Diameter: approx. 10cm; Height: approx. 10cm; Weight: 937g ± 1g)

(12.1) Socket with pin for supply lead (11).

(12.2) On/off switch for fan.

(12.3) Socket to connect the additional electrode (18) which, always carry recording voltage independent of the setting of switch (12.4).

(12.4) On/off switch for recording-voltage on centre electrode (12.5).

(12.5) Centre electrode. Recording is made with on/off switch (12.4) und gleichzeitig gedrücktem Taster (8)

(13) Additional weight (2x) for puck (12) (Weight: 501g ± 1g).

(14) Spring-type elastic ring (2x) for puck (12) serving as holder for an additional peripheral electrode (18) (Weight: 61g ± 1g)

(15) Inelastic ring (2x) for puck (12) with 3 holders for an additional peripheral electrode (18) (Weight: 60g ± 1g)

(16) Dual ring to couple two pucks (12) with three holders for additional electrodes; one holder shift-able (axis of inertia), 2 holders fixed (periphery) (Weight: 120 g ± 1 g).

(17) Rubber band (approx. 3 m) for elastic coupling of two pucks and for elastic limitation of the experimentation surface.

(18) Additional electrode (2x) for insertion into the holders (14), (15) and (16), used as peripheral electrode or centre-of-gravity electrode; with cable and plug for connection to socket (12.3).

(19) Stand base with fastening ring (axis of rotation for experiments on circular motion).

(20) Deflection pulley (337 464 / 337 463) for attachment to clamping strip (6) (for acceleration experiments).

(21) 2 blocks for base (1), height 1cm and 2cm, ∅ 3cm, used to incline the table by approx. 1°, 2° and 3° (inclined plane).

(22) Cord to connect the puck with an accelerating mass (via deflection pulley (20)) or by means of the ring on the axis of rotation of stand base (19).

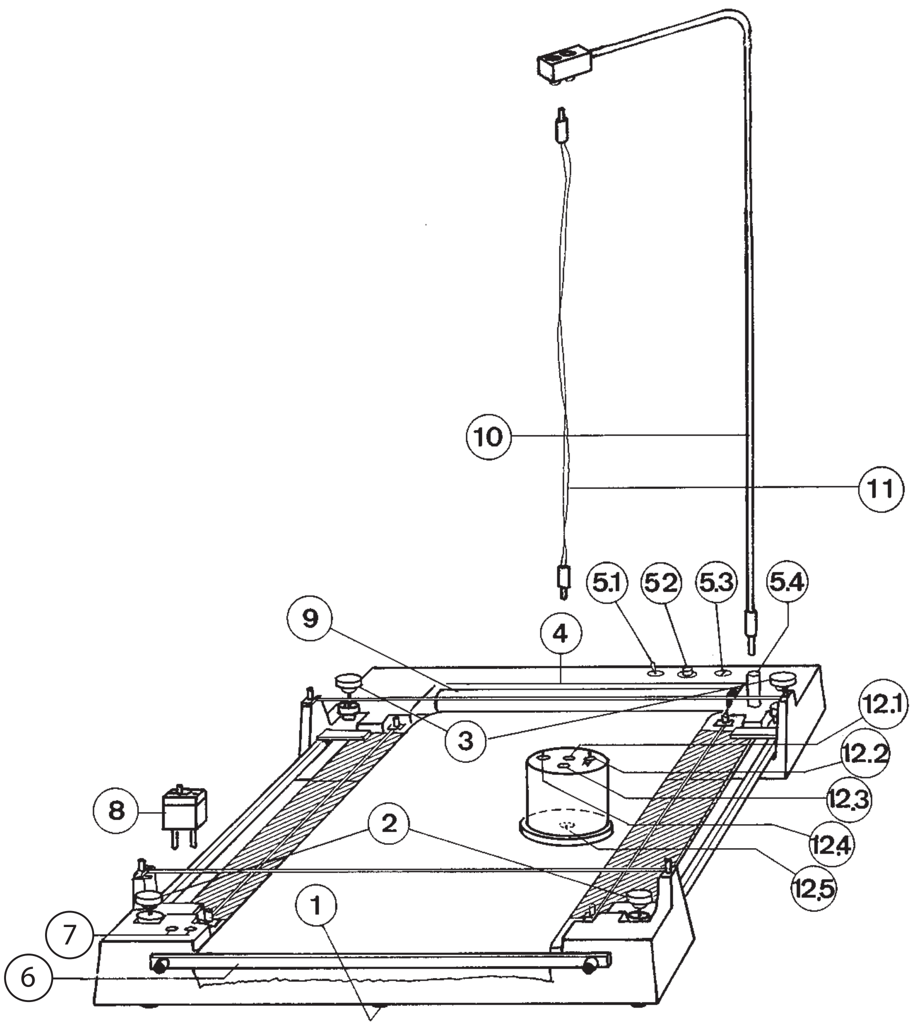

Figure 1.1: Experimental set-up for the air table.

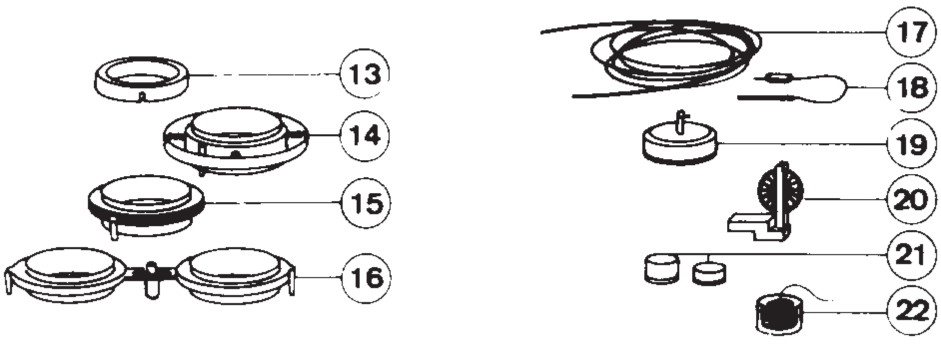

Figure 1.2: Accessories to Air Table.

1.3.2 Experimental Set-up and Adjustment)

Horizontal Adjustment

- Raise the additional feet with adjusting screws (2) until the table only stands on base (1) and the two feet adjustable by levelling screws (3) (three-point support).

- Place a puck on the table at about its centre and connect it via power lead (11) with the power supply arm (10).
- Depress the power switch (5.2) and switch on the fan with switch (12.2) to produce the air cushion.
- Align the glass plate horizontally by means of levelling screws (3) so that the puck does not move.
- Fix the levelling screws (3) by lock nuts.
- Then slowly turn adjusting screws (2) for the additional feet until they touch the work surface lightly without impairing the previously adjusted horizontal position of the table (the puck must remain at rest). Also use lock nuts to fix the screws (2).

Preparing the Pucks

Important: When fitting additional parts to the pucks, this should not be done on the air table.!

- Place the puck (12) (without power lead (11)) on a clean surface (e.g. sheet of paper).;
- Depending on experimental conditions, slip additional weight (13) and/or elastic ring (14) or inelastic ring (15) or the dual ring (16) over the puck and turn the latter so that the stop cam at the puck bottom engages in the groove of the ring.
- When simultaneously using the additional mass (13) and the elastic ring (14) (or the inelastic ring (15) or the dual ring (16)) always fit the additional mass first.
- If required, insert the electrode (18) into the respective holder and connect it to socket (12.3).

Important: Always hold the puck (12) by its body and not by the additionally mounted parts, in order not to change their defined position.

Recording

- Depress power switch (5.2).
- Set frequency selector (5.1) to 50Hz (for marks at time intervals of 0.02 s) or to 10 Hz (for very slow motions or to simplify evaluation).
- Start the fan with switch (12.2).
- For recording by means of the centre electrode (12.5) close switch (12.4); Open switch (12.4) if recording is only to be made by means of the additional electrode (18).
- Set puck into motion and start recording by depressing key switch (8).

If recording does not work after pressing key switch (8) recheck if the metallized recording paper is contacted electrically to the clamping strip (6), optimize if needed.

1.3.3 Tasks for Evaluation

a) Inclined Plane - Acceleration from Slope Force

Component	Quantity
Puck	1
Additional weight	1
Spring-type elastic ring	1
Block for base 1cm	1
Block for base 2cm	1

Execution

- Adjust the air table horizontally (without the additional feet).
- Set the frequency selector to 50Hz.
- Mount the blocks under the feet of the air table on one side, to create a inclined plane.
- Measure the angle of the inclined plane α.
- Place the puck at the raised end of the plane.
- Start the fan, still holding the puck in place.
- Start the recording and let the puck go simultaneously, without adding some momentum.
- Stop the recording when the puck reaches the other end of the table.
- Mount the other blocks to create a different incline and repeat the measurement.

Tasks for Evaluation

- Measure some distances between the recorded marks.
- Calculate the velocity and the momentum at each measured point.
- Now display the measurements (for both inclined planes) in a time versus distance plot and a time versus momentum plot. (You should have 4 plots)
- Do a fit on this data and calculate the acceleration a from the parameters of these fits
- Compare your values to the literature value of $g = 9.81$ m/s

b) Elastic Collision

Component	Quantity
Puck	2
Additional weight	1
Spring-type elastic ring	2

Execution

- Adjust the air table horizontally.
- Set the frequency selector to 50Hz.
- Assemble two pucks with a spring-type ring.
- Connect the puck and start the fan.
- Set both pucks diagonally in motion towards each other.
- Start the recording simultaneously.

Tasks

- Set $v_2 = 0$. Now chose the masses m_1 and m_2 appropriately so that a complete momentum-transfer can happen, meaning after the collision $v_1' = 0$ and the second puck has all the energy. Does this also work if $v_2 \neq 0$? Why? (If you can explain it you don't have to do a measurement, if not, do one and check).
- Is it possible that $v_1' > v_1$? If so, what conditions have to be fulfilled? Make a measurement and check.
- What is the maximum velocity that a puck at rest ($v_2 = 0$) can gain through an elastic collision? What are the conditions for this? Demonstrate it.

Tasks for Evaluation

- Give the conditions for (ii) and (iii) of the tasks and name possible reasons why your measurements where good or bad.
- For all the measurements check the conservation of momentum and energy.

c) Inelastic Collision

Component	Quantity
Puck	2
Additional weight	2
Inelastic ring	2

Execution

- Adjust the air table horizontally.
- Set the frequency selector to 50Hz.
- Assemble both pucks with an inelastic ring.
- Connect the puck and start the fan.
- Start the recording simultaneously.

Tasks

- Solve equation 1.9 with 1.11 for ΔE. What is ΔE? Is the energy conserved in an inelastic collision? Why? (Why not?) Where does the energy go? Make a measurement and show that the energy is or is not conserved.
- Is it possible that $v' = 0$? If so, what conditions have to be fulfilled. Demonstrate it in a measurement.

Tasks for Evaluation

- For task (i) calculate the theoretical ΔE and compare it with the measured one.
- For all measurements check the conservation of momentum and energy.

1.4 Literature

- Friedhelm Kuypers, *Klassische Mechanik*, WILEY-VCH
- Goldstein, Poole & Safko, *Classical Mechanics*, Addison-Wesley