IM2

Modul Mechanics

Air Table

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1.1 Preliminary Questions

- What are the fundamental conservation laws of physics? To which quantities do the apply under what conditions?
- What is a inertial system?
- What is a conservative force?
- What does the NOETHER'S-THEOREM state?
- Deduce the conservation of momentum from NEWTON'S LAWS OF MOTION.
- What is a centre-of-momentum frame and how do you calculate is centre of mass?

1.2 Theory

1.2.1 Newton's Laws of Motion

The principles of motion, noted by ISAAC NEWTON in 1687, called *Newton's laws of motion* build the foundation of classical mechanics. Although, within scope of modern physical theories like quantum mechanics or relativity theories, they are not unconditionally valid, but can still give acceptable prediction within a vast scope of applications.

Newton's First Law - *Lex Prima*

Newton's law first law, also called PRINCIPLE OF INERTIA, describes the motion of physical bodies within an inertial reference frame, in absence of external forces. It declares, that a body in a state of uniform translation of at rest does not change its condition, as long as there are no additional forces applied.
Within those conditions, the body's velocity, its magnitude and its direction are constant. To change its state of motion, a external force (e.g. a gravitational force) has to be applied.

Newton's Second law - *Lex Secunda*

Newton's law second law provides the foundation for most equations of motion in classical mechanics. It states that the rate of change of momentum of a body, is directly proportional to the force applied and this change in momentum takes place in the direction of the applied force. In mathematical terms this correlation is described as

$$\dot{\vec{v}} \propto \vec{F}$$

and was stated in Newton's original work in its universal formulation

$$\vec{F} = \dot{\vec{p}} \,.$$

Since 1750, the following form

$$\vec{F} = m\vec{a} \tag{1.1}$$

stated by Leonhard Euler is known as *Fundamental equation of mechanics*, where $\vec{a}$ describes a change of velocity in time, also known as *acceleration*.

Newton's Third Law - *Lex Tertia*

Newton's third law, the INTERACTION PRINCIPLE, states that to every action there is always opposed an equal reaction. A body 1 that forces an action upon a body 2, experiences the same force, but in opposite direction:

$$\vec{F}_{1\to2} = -\vec{F}_{2\to1}$$

Hence, in a closed system the sum of all forces are equal to zero:

$$\sum_i \vec{F}_i = 0 \tag{1.2}$$

Newton's third law is also known as the principle of *actio and reactio*. Although the requirement of a long range implies, that it loses its general validity within the theories of time dependent electrodynamics and in special relativity and applies only under certain conditions.

Newton's Fourth Law - *Lex Quarta*

Newton's fourth law was only accounted for as an addition in his original work and became *lex quarta* later on. It describes the principle of uninterrupted superposition and states the net response of several forces on one point or rigid body is the sum of all individual forces:

$$\vec{F}_{res} = \sum_i \vec{F}_i \tag{1.3}$$

Conservation of Momentum

The conservation of momentum is one physics fundamental laws and states, that the total momentum in a closed system is preserved or constant. This law in independent from the law of conversation of energy ,and it is also valid for theories like classical mechanics, quantum mechanics and special relativity. For collision processes this indicates, that total momentum before and after a collision must be equal. This is valid for elastic (when kinetic energy is preserved during the collision) as well as for inelastic collisions (when kinetic energy is lost during the collision).
The conservation of energy is a direct implication from Newton's second and third law. Since the force acting upon a body is equal to the change in time of the momentum (Newton's second law):

$$\vec{F} = \dot{\vec{p}}$$

and because there is an equal, opposing force to every force in this system (as long as there is no external force[1]), the sum of all forces is zero. Since this is true for all forces, this also implies that the sum of all vectors, acting in this system is equal to zero. And therefore the sum of all changes in time of all momentums:

$$\vec{F} = \sum_i \vec{F}_i = \sum_i \dot{\vec{p}}_i = \dot{\vec{p}} = 0. \tag{1.4}$$

Since the time derivative of the momentum vanishes, the momentum itself is constant and therefore the centre of mass moves with constant velocity. This leads to the conclusion, that the centre of mass of a system moves with constant velocity and direction, as long as external forces are absent.

[1]For the conservation of momentum, the requirement of no external forces is not complete necessary. It is sufficient to claim, that the sum of all external forces is equal to zero $\sum_i \vec{F}_i^{\text{ext}} = \vec{F}^{\text{ext}} = 0$. Therefore, the individual forces do not have to vanish, but only the sum of all external forces.

Conservation of Energy

In Newton's mechanics, the total energy E of a system consists the sum of kinetic energy T and potential energy V. And in case of bodies moving in a conservative field, this total energy is conserved. Whereas the vector of the force $\vec{F}$ is equal to the negative gradient of the potential energy:

$$\vec{F} = -\vec{\nabla} V$$

A particle moving through a conservative field, in time t, on an arbitrary path $x(t)$, will always do the same work, defined by the difference in potential energy of its start and end point.

With Newton's first law (Eq.: 1.2.1) we can state the following:

$$m\ddot{\vec{x}} = \vec{F} = -\vec{\nabla} V.$$

Multiplying both sides with $\dot{\vec{x}}$ will give us:

$$\begin{aligned} m\ddot{\vec{x}}\dot{\vec{x}} &= -(\vec{\nabla} V)\dot{\vec{x}} \\ &= -\sum_{i=1}^{3} \frac{\partial V}{\partial x_i}\frac{dx_i}{dt} \\ &= -\frac{dV}{dt} \end{aligned}$$

Integration of this term over time leads to the work needed along an arbitrary, continuously differentiable path with the potential energy V_1 at start point and V_2 at end point:

$$\begin{aligned} \int_{t_1}^{t_2} m\ddot{\vec{x}}\dot{\vec{x}}dt &= -\int_{V_1}^{V_2} dV \\ T_2 - T_1 &= -V_2 + V_1 \\ T_1 + V_1 &= T_2 + V_2 \end{aligned}$$

Thus, the sum of potential and kinetic energy stays the same before and after the body was moving, hence the total energy is *conserved*.

Centre-of-Mass Theorem

The centre-of-mass theorem states, that the centre of mass of a multi-body system acts like a point-mass, which has the combined mass of all bodies in this system and is affected by the sum of all force-vectors acting on all the individual bodies.
Hence the centre of mass is moving unaffected of all the inner forces acting between the individual bodies. If all external force-vectors add up to zero, the centre of mass is moving linear, free of forces and without any change in velocity (Newton's first law).

1.3 Experiment

1.3.1 Accessories

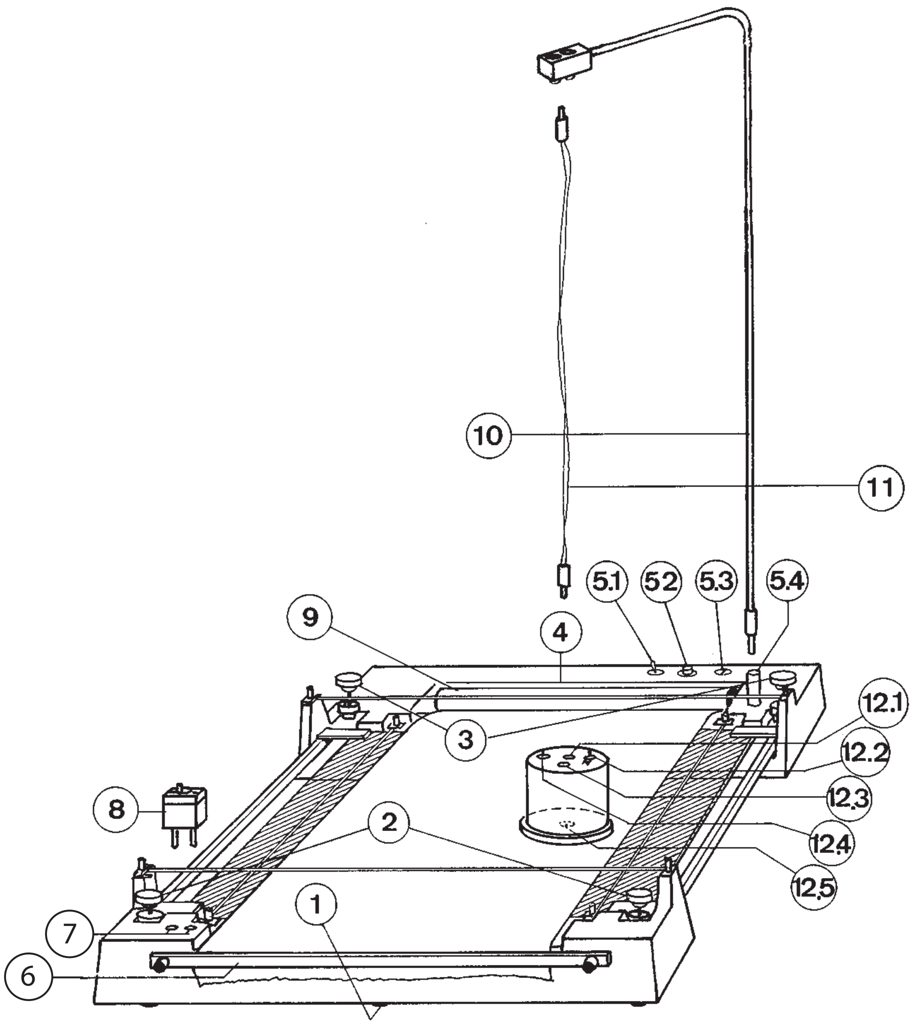

Figure 1.1: Experimental set-up for the air table.

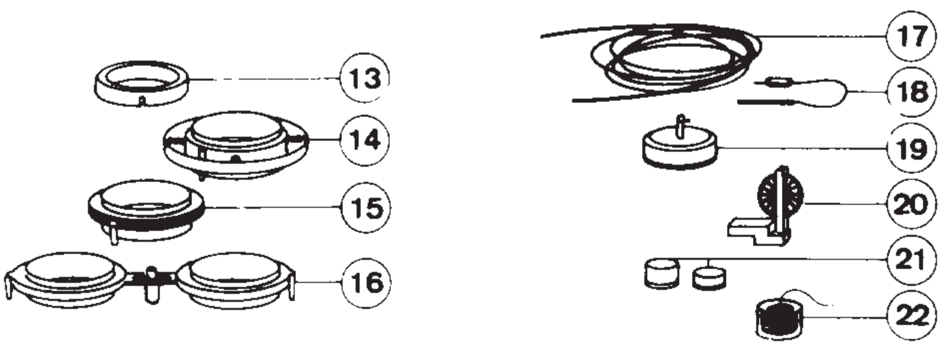

Figure 1.2: Accessories to Air Table.

1 Stable base of the three-point support.

2 Adjusting screws for additional feet (to stabilize the level adjustment obtained with the three-point support 1 / 3)

3 Levelling screws for the three-point support

4 Recess for metallized recording paper

5 Power supply for puck fan and recording electrodes

5.1 Frequency selector (10 Hz/50 Hz) for applying spike pulses to recording electrodes 12.5 and/or 18.

5.2 Power switch with mains indicator lamp.

5.3 Holder with primary fuse.

5.4 Socket for power-supply arm 10.

6 Clamping strip for metallized recording paper and for providing electrical contact (recording circuit).

7 4-mm-sockets, internally connected with clamping strip 6 and power supply (recording circuit).

8 Key switch for switching the registration pulses on and off.

9 Roll of metallized recording paper, 20m length, 45cm width (consumable material).

10 Power supply arm, pluggable into socket 5.4; with two sockets connected in parallel to connect the supply lines 11 for two pucks 12.

11 Power lead (2x), approx. 85 cm long, for voltage supply from the power supply 5 to the pucks 12.

12 Puck (2x) with fan for producing the air cushion and with centre electrode lightly dragging on the recording paper. (Diameter: approx. 10cm; Height: approx. 10cm; Weight: 937g $\pm$ 1g)

12.1 Socket with pin for supply lead 11.

12.2 On/off switch for fan.

12.3 Socket to connect the additional electrode 18 which, always carry recording voltage independent of the setting of switch 12.4.

12.4 On/off switch for recording-voltage on centre electrode 12.5.

12.5 Centre electrode. Recording is made with on/off switch 12.4 und gleichzeitig gedrücktem Taster 8

13 Additional weight (2x) for puck 12 (Weight: 501g $\pm$ 1g).

14 Spring-type elastic ring (2x) for puck 12 serving as holder for an additional peripheral electrode 18 (Weight: 61g $\pm$ 1g)

15 Inelastic ring (2x) for puck 12 with 3 holders for an additional peripheral electrode 18 (Weight: 60g $\pm$ 1g)

(16) Dual ring to couple two pucks (12) with three holders for additional electrodes; one holder shift-able (axis of inertia), 2 holders fixed (periphery) (Weight: 120 g $\pm$ 1 g).

(17) Rubber band (approx. 3 m) for elastic coupling of two pucks and for elastic limitation of the experimentation surface.

(18) Additional electrode (2x) for insertion into the holders (14), (15) and (16), used as peripheral electrode or centre-of-gravity electrode; with cable and plug for connection to socket (12.3).

(19) Stand base with fastening ring (axis of rotation for experiments on circular motion).

(20) Deflection pulley (337 464 / 337 463) for attachment to clamping strip (6) (for acceleration experiments).

(21) 2 blocks for base (1), height 1cm and 2cm, ∅ 3cm, used to incline the table by approx. 1°, 2° and 3° (inclined plane).

(22) Cord to connect the puck with an accelerating mass (via deflection pulley (20)) or by means of the ring on the axis of rotation of stand base (19).

1.3.2 Experimental Set-up and Adjustment)

Horizontal Adjustment

- Raise the additional feet with adjusting screws (2) until the table only stands on base (1) and the two feet adjustable by levelling screws (3) (three-point support).
- Place a puck on the table at about its centre and connect it via power lead (11) with the power supply arm (10).
- Depress the power switch (5.2) and switch on the fan with switch (12.2) to produce the air cushion.
- Align the glass plate horizontally by means of levelling screws (3) so that the puck does not move.
- Fix the levelling screws (3) by lock nuts.
- Then slowly turn adjusting screws (2) for the additional feet until they touch the work surface lightly without impairing the previously adjusted horizontal position of the table (the puck must remain at rest). Also use lock nuts to fix the screws (2).

Preparing the Pucks

Important: When fitting additional parts to the pucks, this should not be done on the air table.!

- Place the puck (12) (without power lead (11)) on a clean surface (e.g. sheet of paper).;
- Depending on experimental conditions, slip additional weight (13) and/or elastic ring (14) or inelastic ring (15) or the dual ring (16) over the puck and turn the latter so that the stop cam at the puck bottom engages in the groove of the ring.

- When simultaneously using the additional mass 13 and the elastic ring 14 (or the inelastic ring 15 or the dual ring 16) always fit the additional mass first.
- If required, insert the electrode 18 into the respective holder and connect it to socket 12.3.

Important: Always hold the puck 12 by its body and not by the additionally mounted parts, in order not to change their defined position.

Recording

- Depress power switch 5.2.
- Set frequency selector 5.1 to 50Hz (for marks at time intervals of 0.02 s) or to 10 Hz (for very slow motions or to simplify evaluation).
- Start the fan with switch 12.2.
- For recording by means of the centre electrode 12.5 close switch 12.4; Open switch 12.4 if recording is only to be made by means of the additional electrode 18.
- Set puck into motion and start recording by depressing key switch 8.

If recording does not work after pressing key switch 8 recheck if the metallized recording paper is contacted electrically to the clamping strip 6, optimize if needed.

1.3.3 Tasks for Evaluation

a) Uniform Linear Motion

Component	Quantity
Puck	1
Additional weight	1

Execution

- Adjust the air table horizontally.
- Set the frequency selector to 50Hz.
- Place the puck in one corner.
- Start the fan with switch.
- Set puck into motion and start recording by depressing key switch 8.

Tasks for Evaluation

- Determine the average velocity of the puck.
- Determine the momentary velocity of the puck at every recorded dot and display your findings in a plot with time versus velocity.
- Display time versus distance of a uniform linear motion in a plot.
- Determine the systematic and statistical error propagation for all your measurements.
- Determine the deficits from friction.

b) Inclined Plane - Acceleration from Slope Force

Component	Quantity
Puck	1
Additional weight	1
Spring-type elastic ring	1
Block for base 1cm	1
Block for base 2cm	1

Execution

- Adjust the air table horizontally (without the additional feet).
- Set the frequency selector to 50Hz.
- Mount the block for base on three-pint support, to create a inclined plane.
- Place the puck at the raised end of the plane.
- Start the fan, still holding th puck in place.
- Start the recording and let the puck go simultaneously, without adding some momentum.
- Stop the recording when the puck reaches the other end of the table.
- Mount the other block for base and repeat the measurement.

Tasks for Evaluation

- Display both measurements in a time versus distance plot and a time versus momentum plot for the accelerated motion.
- Calculate the average acceleration force.
- Compare your values to the calculated, theoretical ones.

c) Inclined Throw - Bomb-Trajectory

Component	Quantity
Puck	1
Additional weight	1
Spring-type elastic ring	1
Block for base 1cm	1

Execution

- Assemble the air table as an inclined plane (see b)).
- Set the frequency selector to 50Hz.
- Assemble the puck with a spring-type elastic ring and additional weight.
- Pay attention to the alignment of the recording paper and the edge of the table. Both have to be parallel, for simple drawing of the X and y axis and further evaluation of the recorded data.
- Start the fan and place the puck on the raised side 1cm from the edge of the paper and 10cm from the rubber band.
- Align two springs of the spring-type ring perpendicular to the rubber bands.
- Let the puck go: It moves linearly downwards and gets reflected by the rubber band. Now its motion describes a parabola. (For practice: place the puck closer to the rubber band and increase the distance in small steps, to avoid damaging the puck).
- Start the recording when the puck is reflected by the rubber band and stop it when the puck has returned to the rubber band.

Tasks for Evaluation

- Dismantle this motion into its horizontal and vertical components by drawing a tangent at the midpoint of the parabola as X axis and a perpendicular one as Y axis.
- Display the motion of the declined throw in a time versus distance plot.
- Determine the type of its horizontal and vertical components.
- Calculate the acceleration of the up- and downwards motion.
- Compare your values to the calculated, theoretical ones.

d) Motion of the Centre of Mass - Superposition of Translation and Rotation

Component	Quantity
Puck	1
Spring-type elastic ring	1
Additional electrode	1

Execution

- Adjust the air table horizontally.
- Set the frequency selector to 50Hz.
- Assemble the spring-type ring to the puck.
- Insert the additional electrode into the holder of the spring-type ring and connect it to the puck.
- Connect the puck and start the fan.

- Set the puck in motion with an additional angular momentum.
- Start and stop the recording a couple of times (this should enable you to distinguish between the recordings of the centre-electrode and the peripheral-electrode, since they are separated in time).
- Stop the recording when the puck reaches the edge of the table.

Tasks for Evaluation

- Determine the centre of mass velocity.
- Determine the angular velocity of the puck.
- Display translation and rotation in a time versus distance plot.
- Determine the type of motion of the centre of mass and of the angular motion.

e) Elastic Collision

Component	Quantity
Puck	2
Additional weight	1
Spring-type elastic ring	2

Execution

- Adjust the air table horizontally.
- Set the frequency selector to 50Hz.
- Assemble two pucks with a spring-type ring and add an additional mass to only one of them.
- Connect the puck and start the fan.
- Set both pucks diagonally in motion towards each other.
- Start the recording simultaneously, interrupt it for a short time to get synchronous measurements.

Tasks for Evaluation

- Determine the conservation of momentum for the elastic collision.
- Determine the momentum transfer between the pucks.
- Determine the conservation of energy.
- Calculate and display the centre of mass of this system and determine its type of motion.

f) Inelastic Collision

Component	Quantity
Puck	2
Additional weight	2
Inelastic ring	2

Execution

- Adjust the air table horizontally.
- Set the frequency selector to 50Hz.
- Assemble both pucks with additional mass and a inelastic ring.
- Connect the puck and start the fan.
- Put one puck at rest and set the other one diagonally in motion towards the one at rest.
- Start the recording simultaneously.

Tasks for Evaluation

- Determine the conservation of momentum for the inelastic collision.
- Determine the momentum transfer between the pucks.
- Determine the conservation of energy.
- Calculate and display the centre of mass of this system and determine its type of motion.

1.4 Literature

- Friedhelm Kuypers, *Klassische Mechanik*, WILEY-VCH
- Goldstein, Poole & Safko, *Classical Mechanics*, Addison-Wesley