IM1

Modul Mechanics

Determination of Volume and Density

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1.1 Questions to prepare

- What are the definitions of solids, liquids, and gases? What are the boundaries and differences and how can they transition into each other?
- What is the relationship between volume, mass, and density in general? Which assumptions have to be made for this relationship to be true?
- What is the definition of buoyancy and what causes it?
- Which quantity can be determined using buoyancy?
- What is an ideal gas and what is the difference when comparing to a real gas?

1.2 Introduction

1.2.1 Volume

THE VOLUME V describes the spacial extend of an object and has the SI-unit m^3. In the case of liquids and gases the *liter* l is commonly used, where $1l=1dm^3$. The volume depends on the temperature and pressure acting on the body.
During history countless methods were used to measure the volume of substances. In the following, we will list the most common ones:

- **Weighing:** When the density of a substance in knows, one can use a scale to determine the mass and derive the volume. In the case of inhomogeneous substances, this approach will only yield a rough estimate.
- **Volumetric measurement:** Another simple but very effective method is to fill a volume with a liquid or sand. Naturally, this procedure is only applicable when investigating hollow objects. After filling the object, the used liquid is measured using a container of known volume.
- **Water displacement:** Similarly, a often useful and versatile strategy is to submerge the object in question under water. The displaced water is then again measured in a container of known volume.
- **Calculating:** If the object has a simple geometrical shape (sphere, cube, cuboid etc.) one can simply calculate the volume, assuming the outer dimensions (diameter, height, width, etc.) are known.

In this experiment, one of the goals will be to determine the volume of different objects. Since these objects are simple shapes without hollow cavities, we will be using the principle of water displacement and calculating from outer dimensions.

1.2.2 Density

The density ρ is a physical constant which characterizes the material an object is made of. The SI-unit is kg/m^3. As a material constant, this quantity depends only on the material and not on the shape or size of the studied object. The density should not be confused with the specific weight. While the latter describes the relationship between weight and volume, density describes the relationship between mass and volume.

In the case of homogeneous objects, where the density ρ is constant everywhere, the mass m is given by the product of ρ and the volume V:

$$m = \rho V \tag{1.1}$$

Is the volume and mass known, then the density can simply be calculated. As mentioned before, the volume can depend on temperature and pressure, therefore this too applies to the density.

1.3 Theory

1.3.1 Determining the density of a liquid using a pycnometer

A pycnometer is a vessel which allows to prepare a precisely repeatable volume of liquids. Therefore, it is particularly suitable to measure the density of liquids by comparison with a reference liquid of known density. Due to the identical volume, it is sufficient to measure the mass of the liquids.

Determination of the density without buoyancy

First, we wish to take a look at the *uncorrected* density of the liquids we want to measure. Here, the following naming convention will be used for the uncorrected masses and densities:

$$\begin{aligned} m_{Pm} &:= \text{Mass of empty pycnometer} \\ m_{Pm+H_2O} &:= \text{Mass of pycnometer filled with water} \\ m_{Pm+Fl} &:= \text{Mass of pycnometer filled with liquid} \\ \rho_{H_2O} &:= \text{Density of water} \\ \rho_{Fl} &:= \text{Density of liquid to be determined} \end{aligned} \tag{1.2}$$

The mass of water is $m_{Pm+H_2O} - m_{Pm}$, the one of the liquid $m_{Pm+Fl} - m_{Pm}$. From this we can derive the uncorrected density from the following equations

$$\begin{aligned} \rho_{H_2O} &= \frac{m_{Pm+H_2O} - m_{Pm}}{v} \\ \rho_{Fl} &= \frac{m_{Pm+Fl} - m_{Pm}}{v} \end{aligned}$$

and due to the equality of the volume v:

$$\rho_{Fl} = \frac{m_{Pm+Fl} - m_{Pm}}{m_{Pm+H_2O} - m_{Pm}} \cdot \rho_{H_2O} \tag{1.3}$$

Determination of the density with buoyancy

In this section, we want to take the effect of buoyancy due to the surrounding air into account. We will still use the same naming convention as above (1.2). Since all masses are measured in air, they have to be viewed as uncorrected. However, the density can be viewed as corrected for buoyancy. We will denote the outer volume of the pycnometer as V, ther internal volume as v.
The equation describing the measured mass m_{Pm+H_2O} will contain the following terms:

$$m_{Pm+H_2O} = \underbrace{v \cdot \rho_{H_2O}}_{\text{corrected mass of the internal volume filled with water}} + \underbrace{(V - v) \cdot \rho_{Pm}}_{\text{corrected mass of the pycnometer}} - \underbrace{V \cdot \rho_{\text{Air}}}_{\text{buoyancy}} \tag{1.4}$$

One should note that $V - v$, the difference between outer and inner volume, is the volume of the glas the pycnometer is made of.
In analogy to equation (1.4), we obtain the following equations for the mass of the pycnometer filled with liquid m_{Pm+Fl} and for the empty (filled with air) pycnometer:

$$m_{Pm+Fl} = v \cdot \rho_{Fl} + (V - v) \cdot \rho_{Pm} - V \cdot \rho_{Air} \tag{1.5}$$

$$m_{Pm} = v \cdot \rho_{Air} + (V - v) \cdot \rho_{Pm} - V \cdot \rho_{Air} \tag{1.6}$$

Rearranging (1.6) according to:

$$(V - v) \cdot \rho_{Pm} = m_{Pm} + (V - v) \cdot \rho_{Air}$$

and using (1.4) and (1.5) yields:

$$m_{Pm+H_2O} = v \cdot \rho_{H_2O} + m_{Pm} - v \cdot \rho_{Air} \tag{1.7}$$

$$m_{Pm+Fl} = v \cdot \rho_{Fl} + m_{Pm} - v \cdot \rho_{Air} \tag{1.8}$$

The equation for the corrected density of the liquid ρ_{Fl} can now be obtained from (1.7) and (1.8) by rearranging:

$$\rho_{Fl} = \underbrace{\frac{m_{Pm+Fl} - m_{Pm}}{m_{Pm+H_2O} - m_{Pm}} \cdot \rho_{H_2O}}_{\text{uncorrected density (1.3)}} \left(1 - \frac{\rho_{Air}}{\rho_{H_2O}}\right) + \rho_{Air} \tag{1.9}$$

1.3.2 Determining the density of solutions assuming known constituents

We will now calculate the density of a solution made of the liquid measured in the previous section and water. Here we will assume that the densities of these constituents are known. We will again begin with an anlalysis neglecting buoyancy and the use the same correction as above afterwards. Since the water will now only make up a part of the internal volume, we will denote this volume as V'_{H_2O} in the following.
The overall uncorrected mass of the solution is then

$$m_L = \rho_{Fl} V_{Fl} + \rho_{H_2O} V'_{H_2O} \tag{1.10}$$

In an ideal solution the volumes of the constituents are simply added together to obtain the overall volume, thus we obtain:

$$\begin{aligned} \rho_L = \frac{m_L}{V_L} &= \frac{\rho_{Fl} V_{Fl} + \rho_{H_2O} V'_{H_2O}}{V_L} \\ &= \rho_{Fl} \cdot \frac{V_{Fl}}{V_{Fl} + V'_{H_2O}} + \rho_{H_2O} \cdot \left(1 - \frac{V_{Fl}}{V_{Fl} + V'_{H_2O}}\right) \\ &= \rho_{Fl} \cdot c + \rho_{H_2O} \cdot (1 - c) \end{aligned} \tag{1.11}$$

where $c = V_{Fl}/(V_{Fl} + V_{H_2O})$ is the volumetric concentration of the liquid in the water solution. Now employing the same correction for buoyancy as above, we arrive at the final expression for the density of the solution:

$$\rho_L = (\rho_{Fl} \cdot c + \rho_{H_2O} \cdot (1 - c)) \cdot \left(1 - \frac{\rho_{Luft}}{\rho_{H_2O}}\right) + \rho_{Luft} \tag{1.12}$$

As we can see, the density depends linearly on the concentration c.

1.3.3 Determining the density of air with a glass bulb

According to Archimedes the buoyancy is equal to the weight of the displaced medium (liquid or gas), but it acts in the opposite direction as gravity. Assuming g the gravitational acceleration on earth and ρ_{Fl} the density of the displaced medium, an object with volume V will experience buoyancy according to the following expression:

$$F_A = g \cdot \rho_{Fl} \cdot V \tag{1.13}$$

In this experiment a glas sphere with an outer volume V_o and inner volume V_i will be used. If the valve of the sphere is open, the pressure inside is the atmospheric pressure p_0. When measured with a scale, the following overall weight is observed:

$$F_G^0 = g \cdot (\underbrace{\rho_g V_g}_{Glas} + \underbrace{\rho_0 V_i}_{Luft} - \underbrace{\rho_0 V_o}_{buoyancy}) \tag{1.14}$$

Where

$$\rho_g V_g := \text{Mass of the glass sphere in vacuum}$$
$$\rho_0 := \text{Density of air at athmospheric pressure } p_0$$

In the next step, the pressure inside the glas sphere is changed to a value p_x. Measuring the weight of the sphere once again, we now obtain:

$$F_G^x = g \cdot (\underbrace{\rho_g V_g}_{Glas} + \underbrace{\rho_x V_i}_{Luft} - \underbrace{\rho_0 V_i}_{buoyancy}) \tag{1.15}$$

For a constant volume and constant temperature, the ideal gas law tells us that:

$$\frac{p_x}{p_0} = \frac{\rho_x}{\rho_0} \tag{1.16}$$

Now subtracting (1.14) and (1.15):

$$F_G^0 - F_G^x = \rho_0 g V_i \left(1 - \frac{p_x}{p_0}\right) \tag{1.17}$$

After some rearranging we finally obtain:

$$\rho_0 V_i \left(\frac{p_0 - p_x}{p_0}\right) = m_0 - m_x \tag{1.18}$$

where $m_0 - m_x$ is the difference in mass at athnmospheric pressure and at pressure p_x.

1.4 Experiment

1.4.1 Materials

Components	Quantity
Qube	2
Sphere	1
Rectangular cuboid	2
Standing vessel	1
Plastic tube	3
Messzylinder	1
Plastic cup 1000ml	1
Measuring cylinder	1
Fishing line	1

Volume and density of solids.

Components	Quantity
Pycnometer	1
Thermometer $-30°C$ - $+110°C$	1
Measuring cylinder 100ml (glass)	2
Ethanol (96%)	1

Density measurements of liquids using a pycnometer.

Components	Quantity
Glass sphere with 2 valves	1
Vacuum pump	1
Cork ring for glass sphere	1

Density of air.

1.4.2 Experimental setup and procedure

Volume and density of solids.

In Fig. 1.1 the experimental setup is shown.

- Pour water in plastic cup *c* (Fig. 1.1)
- Place standing vessel *a* in such a way that the overflowing water is collected in the measuring cylinder *b*.
- Pour water from the plastic cup *c* into the standing vessel *a* until the water is overflowing into the measuring cylinder *b*.
- As a last step, empty the measuring cylinder *b* and place it back under *a* before starting the measurement.

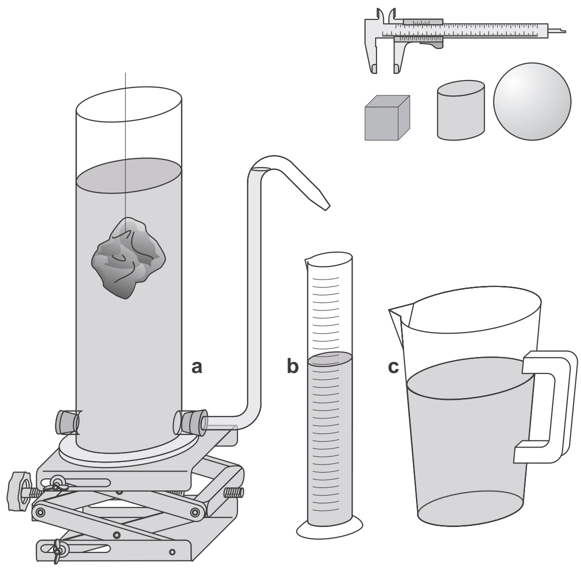

Abbildung 1.1: Experimental setup, volume and density of solids.

- Measure the lateral dimensions with the calliper, do so for three different positions for each dimension.
- Measure the weight of the wooden cube with the precision scale.
- Place the wooden cube into the standing vessel *a* and push it under water using the metal wire until it is completely submerged. Measure the amount of displaced water using the measuring cylinder *b*.
- Repeat these measurements with the remaining objects, to this end, always refill the water in the standing vessel *a* and empty the measuring cylinder for each measurement.

Density measurements of liquids using a pycnometer

The experimental setup is shown in Fig. 1.2.

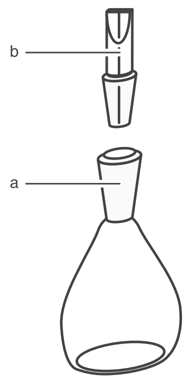

Abbildung 1.2: Experimental setup. Density measurements of liquids using a pycnometer.

- Measure the weight of the empty and dry pycnometer.

$T[°C]$	$\rho[g/cm^3]$	$T[°C]$	$\rho[g/cm^3]$
15	0.999099	23	0.997540
16	0.998943	24	0.997299
17	0.998775	25	0.997047
18	0.998596	26	0.996785
19	0.998406	27	0.996515
20	0.998205	28	0.996235
21	0.997994	29	0.995946
22	0.997772	30	0.995649

Tabelle 1.1: Density of water as a function of temperature.

$T[°C]$	$\rho[g/l]$
10	1.2466
15	1.2250
20	1.2041
25	1.1839
30	1.1644
35	1.1455

Tabelle 1.2: Density of air as a function of temperature.

- Fill the pycnometer a (Fig. 1.2) until its neck is filled to one third, wait until all gas bubbles escaped, carefully insert the plug b (Fig. 1.2) so that the excess water can escape through the bore. This bore should be filled with water when the pycnometer is closed.
- Measure the weight of the pycnometer filled with water.
- Measure the water temperature using the thermometer and determine the density of water using Tab. 1.1.
- Determine the air temperature using the weather station and determine the density of air using Tab. 1.2.
- Repeat the measurement with pure ethanol as well as five different ethanol-water solutions.
- Take care to clean the pycnometer for each measurement, especially for solutions, small gas bubbles can take some time to escape before closing the pycnometer with the plug.
- Measure and not down the weight of the pycnometer as well as the volumetric concentration c of your solution.

To make a ethanol-water solution with a target volumetric concentration c of the ethanol, use the following procedure:

- Fill measuring cylinder 1 with $V_1 = c \cdot 100$ml of ethanol.
- Fill measuring cylinder 2 with a volume $V_2 = (1 - c) \cdot 100$ml of water.
- Combine both liquids in one measuring cylinder, do so carefully to avoid the creation of gas bubbles.

Important: To not waste material, use only one batch of ethanol. By continuously reducing the concentration of the solution by adding additional water, you can reuse the liquid. Plan in advance how to proceed with this measurement to obtain reasonable concentrations.

Important: Only handle ethanol and ethanol-water solutions in glass containers. Plastic containers can be damaged when exposed to ethanol.

Density of air

The experimental setup is shown in Fig. 1.3.

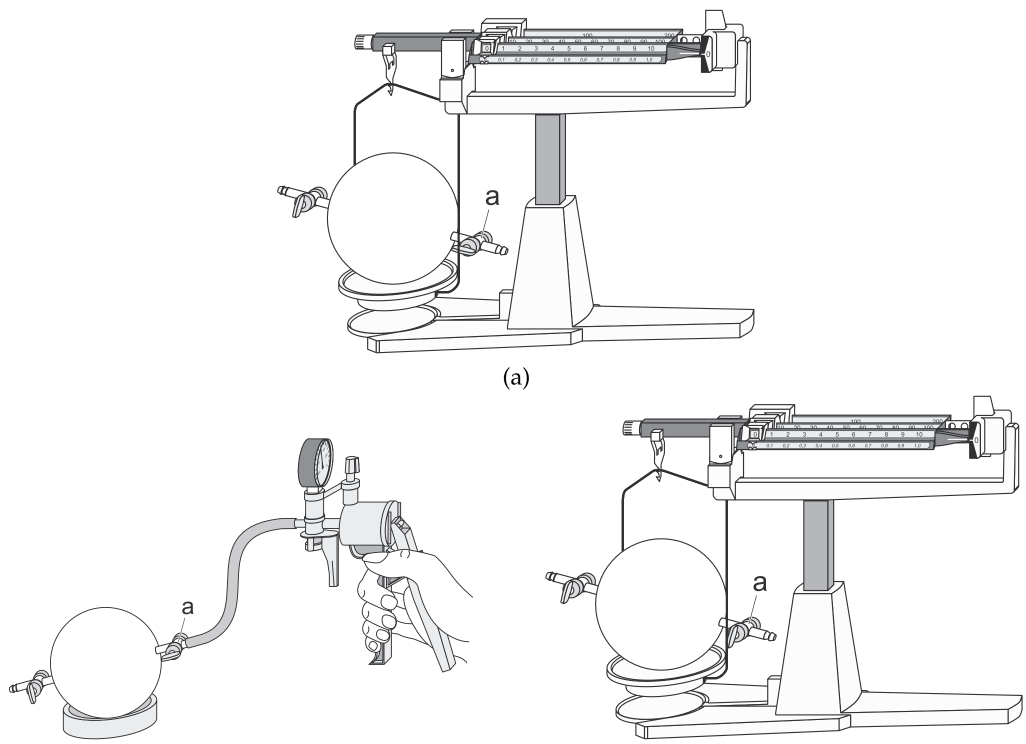

Abbildung 1.3: Experimental setup. Density of air.

Important: The valves of the glass sphere are very fragile. Great care has to be taken when handling the glass sphere, or when connecting/disconnecting the hand pump.

- Measure the weight of the glass sphere using the precision scale, do so while opening one of the valves to measure at atmospheric pressure (see Fig. 1.3 (a)).
- Connect the hand pump and reduce the pressure in the glass sphere as much as possible. The pressure indicated on the hand pump shows the pressure difference Δp tot he surrounding atmospheric pressure. (see Fig. 1.3 (b)). This current atmospheric pressure can be obtained from the weather station.

- Close the valve *a* and carefully remove the hand pump (Fig. 1.3 (c)).
- Measure the weight of the glass sphere again with this internal vacuum.
- Reconnect the hand pump and open valve.
- Carefully let a little bit of air enter the glass sphere to increase the pressure.
- Close the valve and disconnect the pump again, before measuring the weight of the glass sphere.
- Repeat this procedure for five different internal pressures.

Clean all components and tidy up your workplace.

1.5 Tasks for experimental analysis

Volume and density of solids

- For *all* objects, determine the volume using the water displacement method.
- Using this volume, calculate the density of the materials with the weights you measured for all objects.
- For the measurements with the calliper, calculate the mean value, the standard deviation as well as the standard deviation of the mean.
- With these dimensions, again calculate a volume and a density for each object/material. Give the results including both the statistical as well as the systematic error.
- Compare the results obtained with these two methods for each object.

Density measurements of liquids using a pycnometer

- Determine the *corrected* density of ethanol and compare your value with literature.
- For all ethanol-water solutions, determine the *corrected* density of the liquid.
- Present a reasonable error estimation for your results.
- Investigate the impact of buoyancy in these measurement. To do this, compare the corrected and the uncorrected density values.
- Investigate the relationship between the density of the solution and the volumetric concentration of ethanol. Record the values in a graph as a function of c and provide a fit of the data.
- Compare the measured values with the theoretically predicted values given by (1.12). To this end, it is convenient to simply plot (1.12) in the same graph for comparison. Explain your observations.

Density of air

- Using the values at atmospheric pressure (m_0, p_0) as reference, present all pairs m_x, p_x in a graph, by plotting $m_0 - m_x$ against $(p_0 - p_x)/p_0$. How can you extract the density of air at atmospheric pressure from this graph?
- Make similar graphs, using each pair m_x, p_x as reference once. extract the density of air ρ_x for all remaining pressures p_x.
- Present your results including a reasonable error estimation.
- Finally, plot the density of air as a function of pressure. Fit the data and add the literature value for the density of air under normal conditions.