

IIW5

Modul Thermodynamics

Thermodynamic Cycles

In this experiment, the functionality of a heat engine will be examined, using a Stirling engine. From the observed pV diagram, the energy converted into work and the efficiency of the heat engine are to be determined.

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1.1 Preliminary Questions

- What are the fundamental theorems of thermodynamics?
- What is a thermodynamic cycle?
- How does a heat pump, and how does a heat engine work?
- What is a Stirling-Motor?
- What is a Carnot-Process?
- How is efficiency defined?
- What is the meaning of isochore, isobar, isotherm and adiabatic?
- What are the most well known ideal gas laws?

1.2 Theory

1.2.1 Basics

In 1816, Robert Stirling filed the patent for a hot-air machine, now known as the Stirling engine. Today, the Stirling engine is mainly used to study the principle of heat engines, since in this case the transformation process of thermal energy into mechanical energy is particularly easy to understand.

The Stirling engine undergoes a process of energy changes in a gaseous medium. Considering the gas as a multitude of particles of disordered motion and considering only collisions of the particles with each other and their enclosure, it can be described by the equation of state of the ideal gas:

$$p \cdot V = n \cdot R \cdot T = N \cdot k_B \cdot T. \quad (1.1)$$

where $n(N)$ is the number of particles (in Mol) in the volume V , at a pressure p and temperature T , R is the universal gas constant ($R = 8.314 \text{ J}/(\text{K}\cdot\text{mol})$) and k_B is the Boltzmann-Constant ($k_B = 1.38 \cdot 10^{-23} \text{ J}/\text{K}$). The Stirling engine consists of a gas contained in a cylindrical enclosure behind a moving piston. Its volume V is thus determined by the position of the piston. By means of an eccentric axis, the periodic changing volume of the gas transforms the piston movement into a rotational movement. The volume changes can be caused by different physical processes (combustion, steam, heat transfer, ...). The state changes of the gas occurring during a cycle are reversible and always return to their original state. Such a process is called CYCLE PROCESS.

1.2.2 Fundamental Theorems of Thermodynamics

If the heat energy ΔQ is added to a system from the outside, it can be partially consumed at a workload $-\Delta W$. The remainder of ΔQ leads to an increase of the system's internal energy U by ΔU :

$$\Delta Q = \Delta U - \Delta W \quad (1.2)$$

Because the system releases energy to the outside, the workload is calculated negatively. The inner energy U can be kinetic energy of the molecules ($dU = C_V \cdot \delta T$), thus leading to a warming; with ideal gases this is the only possible form of internal energy. If we are dealing with a gas, the external workload is done only by pressure work $\Delta W = -p \cdot \Delta V$. ΔW is calculated negatively because work is done only when the system loses energy and ΔV is positive.

Application of the first Law to the ideal Gas

Since there are no forces acting between the molecules in an ideal gas, its internal energy U is only a function of T

$$U = U(T) \quad (1.3)$$

The differential dU then results in

$$dU = \left. \frac{\partial U}{\partial T} \right|_{T_0} \cdot dT. \quad (1.4)$$

Experience shows that $\left. \frac{\partial U}{\partial T} \right|_{T_0}$ is constant over large parts:

$$C_V = \left. \frac{\partial U}{\partial T} \right|_{T_0} = \text{constant}. \quad (1.5)$$

1.2.3 Fundamental Processes in an Ideal Gas

1. The isotherme process: We have an isothermal process if and only if T is constant. Because of $dU = n \cdot C_V \cdot dT$ and $dT = 0$ it follows from the first law that:

$$dQ = -dW = p \cdot dV \quad (1.6)$$

With the equation of state of the ideal gas (1.1) follows:

$$dQ = \frac{n \cdot R \cdot T}{V} dV \quad (1.7)$$

Integration leads to:

$$\begin{aligned} \int_1^2 dQ &= n \cdot R \cdot \int_1^2 \frac{1}{V} dV \\ Q &= n \cdot R \cdot T \cdot \ln \left(\frac{V_2}{V_1} \right) \end{aligned} \quad (1.8)$$

2. The adiabatic Process: A thermodynamic process is called adiabatic if none of the systems heat is released to the outside during the process. In reality, this means that the process is much faster than thermal diffusion. So in the adiabatic process we assume that Q is constant and thus dQ null. From the first law follows:

$$\begin{aligned} dU &= dW = -pdV \\ dU &= nC_V dT \end{aligned} \quad (1.9)$$

and therefore:

$$nC_V dT = -n \cdot R \cdot T \frac{dV}{V} \quad (1.10)$$

again, from integration follows:

$$\begin{aligned} \int_{T_1}^{T_2} \frac{dT}{T} &= -\frac{R}{C_V} \int_{V_1}^{V_2} \frac{dV}{V} \\ \ln \left(\frac{T_2}{T_1} \right) &= -\frac{R}{C_V} \ln \left(\frac{V_2}{V_1} \right) \end{aligned} \quad (1.11)$$

With the Definition of the adiabatic exponent $\gamma = C_p/C_V$ it follows:

$$\frac{R}{C_V} = \frac{C_p - C_V}{C_V} = \gamma - 1$$

$$\ln\left(\frac{T_2}{T_1}\right) = \ln\left(\frac{V_1}{V_2}\right)^{\gamma-1} \quad (1.12)$$

and finally:

$$T_1 \cdot V_1^{\gamma-1} = T_2 \cdot V_2^{\gamma-1} = \text{constant} \quad (1.13)$$

1.2.4 The Carnot Cycle

The CARNOT- Cycle is a reversibly guided process consisting of two adiabatic and isothermal sub-processes:

1. Isotherme Expansion:

$$dU = 0 \quad dQ = -dW_1$$

$$Q_1 = n \cdot R \cdot T_1 \cdot \ln\left(\frac{V_2}{V_1}\right) \quad (1.14)$$

2. Adiabatic Expansion:

$$dQ = 0 \quad dW = dU$$

$$W_2 = C_V(T_2 - T_1)$$

$$= -C_V(T_1 - T_2) \quad (1.15)$$

3. Isotherme Compression:

$$dU = 0 \quad dQ = -dW_3$$

$$Q_2 = n \cdot R \cdot T_2 \cdot \ln\left(\frac{V_4}{V_3}\right) \quad (1.16)$$

4. Adiabatic Compression:

$$dQ = 0 \quad dW = dU$$

$$W_4 = C_V(T_1 - T_2)$$

$$= -C_V(T_2 - T_1) \quad (1.17)$$

According to the first law, the work delivered by the system is thus:

$$-W_{tot} = -W_1 - W_2 - W_3 - W_4$$

$$= Q_1 + Q_2$$

$$= n \cdot R \cdot T_1 \cdot \ln\left(\frac{V_2}{V_1}\right) + n \cdot R \cdot T_2 \cdot \ln\left(\frac{V_4}{V_3}\right) \quad (1.18)$$

From $T \cdot V^{\gamma-1} = \text{constant}$, the adiabatic equation of state follows now:

$$T_1 \cdot V_2^{\gamma-1} = T_2 \cdot V_3^{\gamma-1}$$

$$T_1 \cdot V_1^{\gamma-1} = T_2 \cdot V_4^{\gamma-1} \quad (1.19)$$

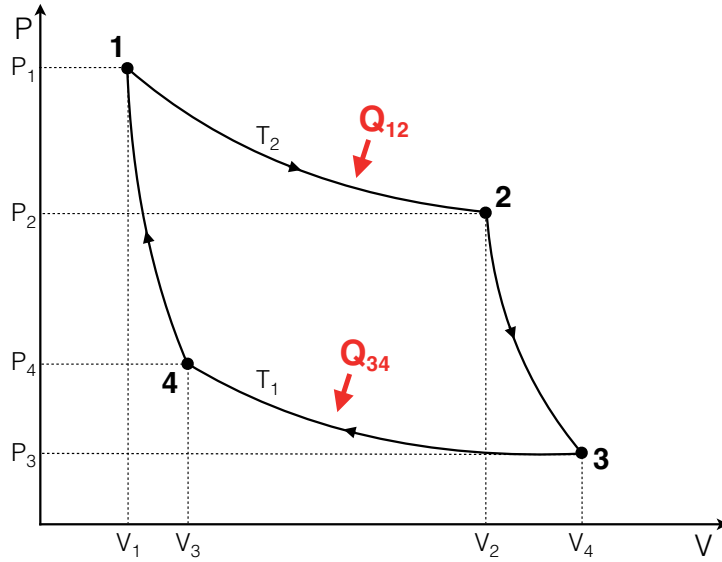


Figure 1.1: pV-Diagram of a Carnot-Cycle.

Dividing the last two equations by one another results in:

$$\begin{aligned} \left(\frac{V_2}{V_1}\right)^{\gamma-1} &= \left(\frac{V_3}{V_4}\right)^{\gamma-1} \\ \left(\frac{V_2}{V_1}\right) &= \left(\frac{V_3}{V_4}\right) \end{aligned} \quad (1.20)$$

With the definition

$$\eta_C = \frac{-W_{tot}}{Q_1} \quad (1.21)$$

this results in Carnot efficiency:

$$\begin{aligned} \eta_C &= \frac{-W_{tot}}{Q_1} = \frac{Q_1 + Q_2}{Q_1} \\ &= \frac{T_1 \ln\left(\frac{V_2}{V_1}\right) + T_2 \cdot \ln\left(\frac{V_4}{V_3}\right)}{T_1 \cdot \ln\left(\frac{V_2}{V_1}\right)} = \frac{T_1 - T_2}{T_1}. \end{aligned} \quad (1.22)$$

1.2.5 The Stirling-Process

In the Stirling process, the adiabatic branches are replaced by two isochoric ones. Since $dV = 0$, and using the first law, this yields in:

The isochore Process

$$dU = C_V \cdot dT \quad (1.23)$$

The calculation for the efficiency of the STIRLING engine η_S can thus be taken literally from η_C :

$$\eta_C = \eta_S \quad (1.24)$$

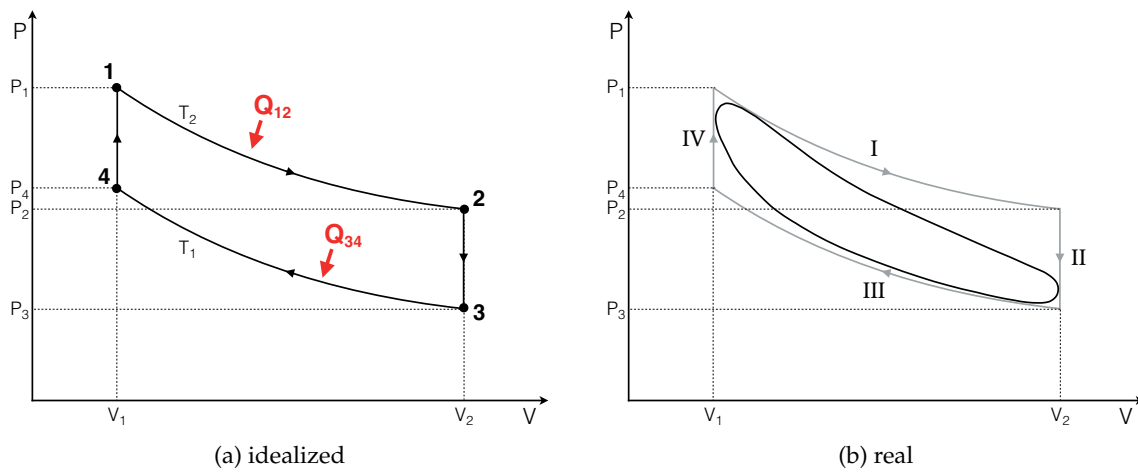


Figure 1.2: pV-Diagram of the Stirling-Cycle.

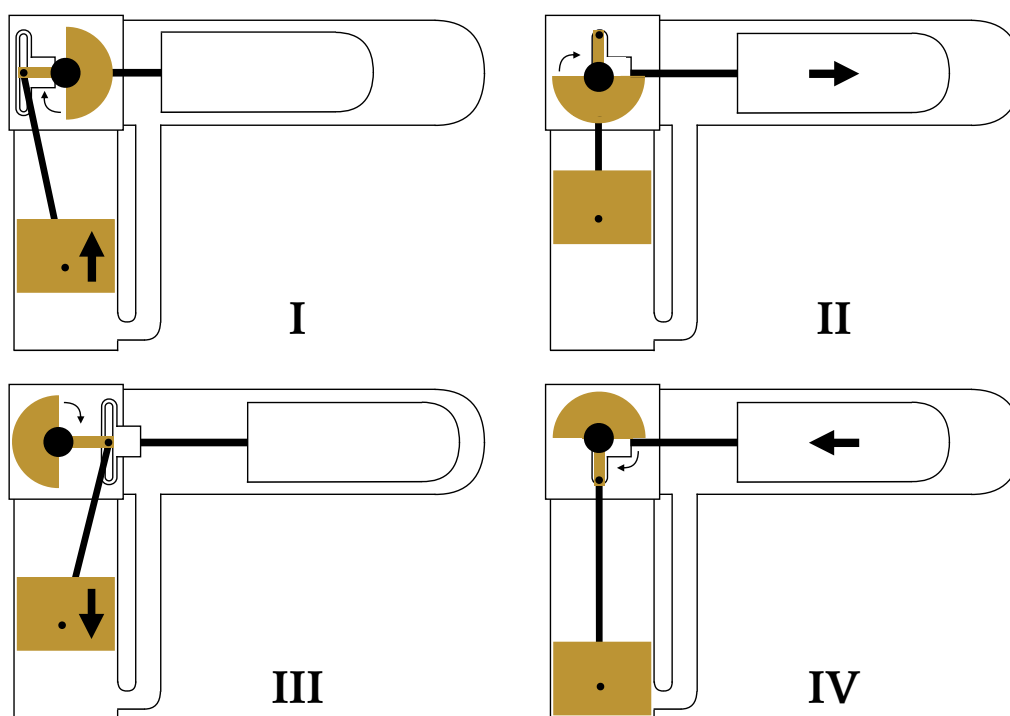


Figure 1.3: How the real Stirling engine works in the 4 phases of the cycle.

1.3 Experiment

1.3.1 Equipment

Component	Quantity
pVnT-Measuring devise	1
Stirling Engine with Sensor Unit	1
Digital Oscilloscope, 40 MHz, 2 Channels	1
Shielded BNC Cable, l = 750 mm	2
Torque Meter	1
Motor/Generator-Unit	1
Variable Resistor 330 Ohm, 1,0 A	1
Thermocouple, NiCr-Ni, -50...1100°C	2
Digital Multimeter	2
chimney for Stirling Engine	1
Beaker Glass 500ml	1
Ethanol, 1l	1
Syringe, 20 ml	1
Cable, 32 A, 500 mm, red	2
Cable, 32 A, 500 mm, blue	3
Measuring Cylinder 50 ml	1

1.3.2 Experimental Setup and Execution

Basic Setup

- Turn on the pVnT-Measuring device.
- Connect the p- and V-outputs of the pVnT measuring device with the Y- and X-channel of the oscilloscope.
- Screw the metal protective cylinder of the two thermocouples.

Calibration of the Measuring Device

For evaluation, the thermodynamic parameters pressure p and volume V are fed as an analogue voltage signal from the pVnT measuring device into an oscilloscope, the temperature T can be read directly on the pVnT measuring device. This means that the device needs to be calibrated first. The procedure is as follows.

- After powering up, the pVnT measuring device displays *cal* (Calibration).
- Fill the beaker with a little water. Now hold both temperature sensors into the water and then press the Calibrate ΔT key on the unit. This only affects the temperature difference between both elements and not the absolute value. The display will now show *ot* (top dead centre). The device acquires both readings and stores their difference until the device is turned off. For the measurement, carefully insert the wire tips of the two thermocouples into the two openings above the glass cylinder. The temperature T_1 must correspond to the temperature at the end of the cylinder (where the burner stands during the measurement). Carefully lay down both cables so that the wires of the thermocouples do not bend suddenly.

- The volume measurement is calibrated by placing the piston (piston made of metal) in the position where the smallest volume is available inside Stirling engine, therefore the piston is at its lowest point. Press the Calibrate V key, when the piston is in the position, described before. The smallest volume (32 cm^3) now corresponds to a voltage of 0 V , the largest volume (44 cm^3) of a voltage of 5 V .
- Incorrect calibration leads to an offset in the volume calculation and thus to a deformation of the pV -diagram. The buttons for the calibration processes have no function during further operation. A new calibration procedure can only be initiated by switching the device off and on.
- The pressure sensor measures the pressure difference against the surrounding air pressure p_0 . To get the absolute value, read off the relative air pressure at the weather station. To evaluate the pV diagram, the pressure output voltage of the meter must now also be calibrated for the existing sensor/instrument combination. Use the syringe and pull the piston out to 20 ml . The measurement can be performed with the oscilloscope: To do this, the oscilloscope is switched on. Check whether the YT mode has been selected (otherwise change into display menu ('Display' button) and press button for mode until 'YT' appears).
The scaling for the channels (CH1 and CH2) should be set to 2 V (otherwise use rotary knobs until 2 V for CH1 & CH2 appears on the bottom left of the display). The time scale should be 5 s (otherwise use rotary knob).
- Now, remove the hose from the bottom plate of the Stirling engine and pull it over the syringe tip. By pushing in the syringe plunger in ml increments up to 10 ml , the pressure is gradually increased. The process is isothermal. Start pushing in the syringe plunger, when the signal line appears on the left edge of the oscilloscope. Stop for each position of the syringe plunger briefly, on the display a kind of 'stairs' should be visible.
- Insert the USB stick into the oscilloscope and save the image directly after completing the overpressure measurement by pressing the 'Print' button (see Fig. A.1.1). The negative pressure area is measured by expansion by pulling the tube over the syringe of 10 ml volume and then increasing the volume by withdrawing the syringe plunger in ml increments up to 20 ml . Proceed analogously as in the pressure range. The volume of the tubing (about 0.07 ml) can be neglected. Figure 1.4 shows a measurement example.
- Now slip the hose over the hose nozzle on the bottom plate near the Stirling engine.

Heat output of the burner

- The alcohol volume is determined before and after the experiment with the scale. The duration of the experiment (burning time of the burner) is measured with a stopwatch

Presentation and Creation of the pV Diagram

- Look at the two voltage signals on the oscilloscope and check that the voltage of the pressure is about 2.5 V and that of the volume is about 0 V .
- Move the two signals vertically to the absolute values "0" to achieve a correct absolute scale (no offset through the oscilloscope).

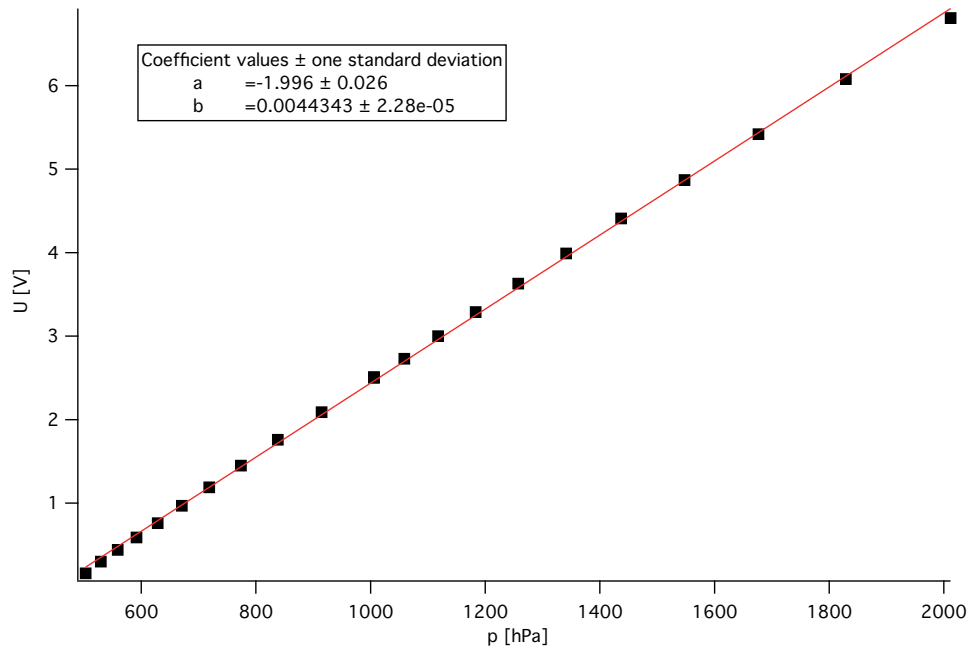


Figure 1.4: Calibration of the pressure output voltage as a function of the pressure p .

- Now set the oscilloscope to XY mode under *Display*.
- Set the resolutions so that a measurement point is at $(2.5/0)$.
- Use the screw to slightly turn the wick out of the opening and ignite it with the lighter. Start the stopwatch.
- Turn the wick down so that only a small flame is visible.
- Place the chimney with the opening towards the Stirling engine over the burner and push it all the way under the engine. Now adjust the height of the flame so that it barely touches the glass cylinder of the engine.
- Now set the upper temperature display to ΔT on the gauge. Now, the lower value indicates the temperature T_2 near the piston, the upper value indicates the difference between this temperature value and the higher temperature value T_1 near the flame.
- Watch the temperature increase and wait until the temperature difference reaches about 100°C .
- Now kick the flywheel clockwise to start the engine. If it does not start, it either needs a bit more momentum, or the temperature difference is still too low. If you are unsure, ask the assistant for help.
- When the engine reaches about 900 rpm , a Stirling cycle should be visible on the oscilloscope.
- Start with the measurements, when the temperatures T_1 , T_2 and the rotation frequency are approximately constant. The lower temperature should be about 70°C .
- To save the image, insert a USB stick into the oscilloscope and press the 'Save / Recall' button. Then the image can be saved on the USB stick by pressing the 'Print' button.
- Stop the stopwatch.

Effective Mechanical Energy

- To load the motor with a certain torque, the scale of the torque meter on the bottom plate and the inner metal piece of the pointer (Prony'sche inclination weight bridle) is mounted on the axis in front of the flywheel.
- The friction between the pointer and the attached piece of metal can be varied with the locking screw. The regulation must be carefully carried out to ensure that the pointer does not begin to oscillate.
- Start the measurements with a small torque.
- Measure about 10-15 different torques (including no load).
- Wait for each change until the torque, rotation frequency, and temperatures are approximately constant before recording the values $\hat{\Delta}$ and the pV graph (save the image to the oscilloscope and note n , T_2 , ΔT per torque).

1.3.3 Tasks for Evaluation

- Create a pU diagram from the pressure sensor calibration and determine the $U - p$ factor from the slope.
In order to create the pU diagram, the respective voltage U for the syringe plunger position must be measured (from the 'Stairs' pictures, see appendix). So for example: $15 \text{ ml} \rightarrow 1.8 \text{ V}$ etc. The syringe volumes must each be converted to the pressure p . Use:

$$p_0 \cdot 20 \text{ ml} = \text{const. (for over pressure range)}$$

$$p_0 \cdot 10 \text{ ml} = \text{const. (for under pressure range)}$$

Where p_0 is the air pressure read at the weather station.

- Also determine the conversion factor $U - V$ from the voltage value U of the minimum and maximum position of the piston ($V = 32 \text{ cm}^3$ equals 0 V , etc.). Using the conversion factors $U - p$ and $U - V$, create a pV diagram for the first oscilloscope image created (without torque load, label the axes with p and V)
- Determine the heat output P_H of the burner. Use:

$$P_H = h \cdot \frac{\Delta m}{\Delta t}$$

Where Δm is the mass difference of the burner and Δt is the measurement time. The calorific value of ethanol h is 26.8 MJ/kg and the calorific value of ethanol is 29.7 MJ/kg .

- From the measured values $\hat{\Delta}$ of the torque measurement, respectively determine the effective mechanical work W_m , the total effective work W_{pV} and the friction work W_r of a cycle (for each used torque) and draw them in a diagram as a function of the rotation frequency (all measured values $\hat{\Delta}$ in a single diagram).

It applies:

$$W_m = 2 \cdot \pi \cdot M \quad W_r = W_{pV} - W_m$$

Where M is the torque applied to the Stirling engine.

- Determine the mechanical P_m from the measurement series and plot it as a function of the rotation frequency n in a diagram. It applies:

$$P_m = W_m \cdot n$$

Where n is the rotation frequency (in Hz/s).

- Explain why the real Stirling cycle is different from the idealized one.
- For 5 points in the middle of the pV diagram, calculate the substance amount n and from this form the mean value as representative substance quantity in the cylinder (use the generated pV diagram). For the calculation, the ideal gas equation is used.
- Calculate the total efficiency η_{tot} of the cycle at maximum mechanical power of the Stirling engine. It applies:

$$\eta_{tot} = \frac{W_m}{W_H} \quad W_H = \frac{P_H}{n}$$

- Also calculate the individual components of the efficiency (Burner efficiency η_H , Carnot efficiency η_C , Inner Efficiency η_{in} and mechanical efficiency $\eta_m \rightarrow$ and the same for the cycle at maximum mechanical power).

$$\eta_H = \frac{|W_1|}{W_H} \quad \eta_C = \frac{T_1 - T_2}{T_1}$$

$$\eta_{in} = \frac{W_{pV}}{|W_t|} \quad \eta_m = \frac{W_m}{W_{pV}}$$

$$W_1 = -\nu \cdot R \cdot T_1 \cdot \ln\left(\frac{V_2}{V_1}\right) \quad W_t = -\nu \cdot R \cdot (T_1 - T_2) \ln\left(\frac{V_2}{V_1}\right)$$

1.4 Literature

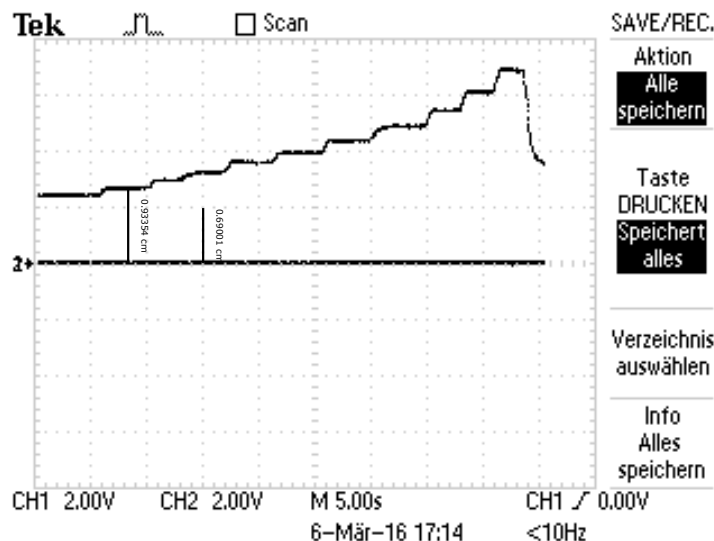
- W. Demtröder, Experimentalphysik 1, "Mechanik und Wärme", Springer Verlag, Berlin
- D. Meschede, "Gerthsen Physik", Springer Verlag, Berlin

A.1 Determination of W_{pV} and conversion factor $U-p$

To measure the oscilloscope images, you can use the software Inkscape as follows.

A.1.1 Conversion factor $U-p$

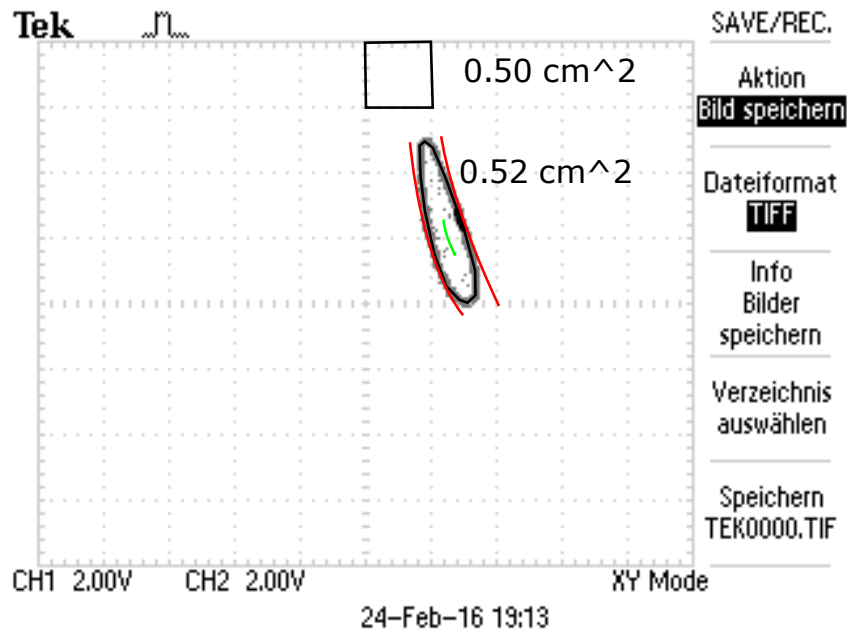
For the conversion factor $U - p$, the 'stairs' image must be measured. Open the Tiff file with Inkscape. Select the Bézier Curve Tool (shortcut: CTRL + F6). Draw a straight line along a square edge (hold down the CTRL key when dragging the line). Double-click to stop the line. Now choose extensions' →, visualize path '→, measure path ...'. In the open dialogue box, select the measurement type 'Length' and the unit of length 'cm'. The font size may need to be adjusted (to approx. 4 – 5 px). The accuracy should be greater than 3. Now click on 'Apply' and then on 'Close'. Now the line was measured. The edge length corresponds to a tension of 2 V. In order to calculate the voltage of the respective syringe volume, the individual steps of the staircase are measured. Draw a line from the stairway to the main axis and measure the lines.



A.1.2 Determination of W_{pV}

To determine W_{pV} , the area of the cycles on the oscilloscope images must be measured. Open the Tiff file with Inkscape. Select the Bézier curve tool (shortcut: CTRL + F6). First draw a line along a square (hold the CTRL key while dragging the line). Start at a corner point of a square and then click on the other three corner points one after the other. Double-click on the starting point to stop the line. Now, 'extensions' →, visualize path '→, measure path ...'. In the opened dialogue box, select the measurement type 'area' and the unit of measure 'cm'. The font size may need to be adjusted (to approx. 4 – 5 px). The accuracy should be greater than 3. The accuracy should now be 'Apply' and then 'Close'. Now the enclosed area was measured.

The area of a square corresponds to a work of: $2 V \cdot$, 'conversion factor $U - V' \cdot 2 V$. 'conversion factor $U - p'$ (in Joule). Now draw a path along the cycle (close the path by double-clicking on the starting point). Measure the surface of the cycle. Now the work W_{pV} can be determined by comparison with the unit square.



For the determination of the number of particles, (approximately) 5 points should be selected on the green line (see example diagram above). The temperature is the average of T_1 and T_2 .