IIW2

Modul Thermodynamics

Gas Laws

In this experiment, a gas thermometer is used to simultaneously measure the pressure p, volume V and temperature T of an enclosed quantity of gas. This way, we can investigate the relationship between the three quantities using air as an example and examine the validity of the ideal gas law $pV = NkT$.

Versuch IIW2 - Gas Laws

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1.1 Preliminary Questions

- What does the equipartition principle say?
- What is absolute zero on the temperature scale?
  What can be observed on the particle level?
- According to the equipartition principle, what is the average energy of a monatomic gas, of a diatomic gas and of a solid?
- What is the Dulong-Petit rule?
- What do Boyle-Mariotte's law, Gay-Lussac's law and Amonton's law say?
  Under what conditions are these three laws valid?
- What dangers are you exposed to during this experiment and how can you best protect yourself? What should you pay particular attention to?

1.2 Theory

1.2.1 The ideal gas as a many-particle system

If there are N molecules of mass m in the cube-shaped volume V, the particle density is ρ=N/V. Now the unit volume contains t_i particles whose x-component of the velocity has the value $|v_x| = |v_i|$. Half of them, $t_i/2$, will fly in the positive x-direction. The cube surface A, which is normal to the x-axis, is then hit by $\frac{t_i}{2} \cdot A \cdot v_i$ molecules per unit time.
Reflection on the wall results in a change in momentum over time or force:

$$F_i = 2 \cdot \frac{t_i}{2} \cdot A \cdot m \cdot v_i^2 \tag{1.1}$$

and thus, the momentum of an individual particle in x-direction is given as

$$P_i = m \cdot t_i \cdot v_i^2 \tag{1.2}$$

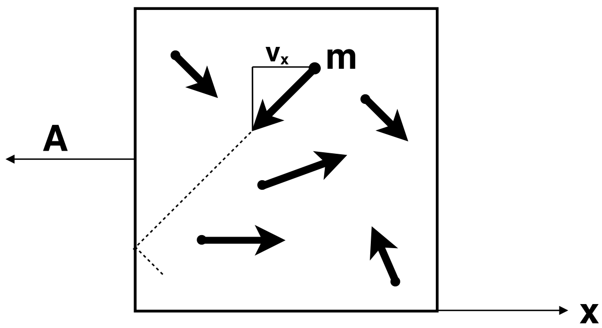

Abbildung 1.1: Darstellung des idealen Gases

We obtain the total pressure by summing over all t_i with velocity components v_i, that is

$$p = m \cdot \sum_i t_i \cdot v_i^2 \tag{1.3}$$

with the mean value

$$< v_x^2 >= \frac{\sum_i t_i \cdot v_i^2}{\sum_i t_i}, \quad \sum_i t_i = t \tag{1.4}$$

follows

$$p = m \cdot \rho \cdot < v_x^2 > \tag{1.5}$$

Because of the molecular disorder, the mean values $< v_x^2 >$,$< v_y^2 >$ and $< v_z^2 >$ must be the same. Thus $< v >^2 = < v_x^2 > + < v_y^2 > + < v_z^2 >$ gives

$$< v_x^2 >= \frac{< v^2 >}{3} \tag{1.6}$$

and results in

$$p = m \cdot t \cdot \frac{< v^2 >}{3} \tag{1.7}$$

By using the average kinetic energy of a molecule

$$< E_{kin} >= \frac{1}{2} m \cdot < v^2 > \tag{1.8}$$

we can define the *basic equation of the kinetic gas theory*

$$p = \frac{2}{3} \cdot \rho \cdot < E_{kin} > \tag{1.9}$$

As a further physical requirement, we need the so-called **equipartition principle**. It bridges the gap between statistical mechanics and phenomenological thermodynamics. Applied to the ideal gas it reads:

$$< E_{kin} >= \frac{3}{2} k \cdot T \tag{1.10}$$

If we express the mean kinetic energy per molecule with (1.10) by the temperature, we obtain from 1.9 the relationship between pressure p, particle density ρ and temperature T for the ideal gas:

$$p = \rho \cdot k \cdot T \tag{1.11}$$

If the volume V contains the amount of substance n mol, the particle density is $\rho = n \cdot N_A / V$ and 1.11 changes to

$$p \cdot V = n \cdot N_A \cdot k \cdot T \tag{1.12}$$

The product $N_A \cdot k$ is the **Universal Gas Constant R**. Subsequently, we obtain the ideal gas law:

$$p \cdot V = n \cdot R \cdot T \tag{1.13}$$

The state of a given quantity of gas can therefore be fully described by three variables: Temperature T, pressure p and volume V. However, these three quantities are not independent of each other. The ideal gas law indicates how these measurable properties depend on each other. In our case, only two of them can be chosen, the third is then uniquely determined. To summarize, the ideal gas law goes as follows:

$$p\, V = N\, k\, T = n\, R\, T \tag{1.14}$$

$$k = \text{Boltzmann constant } (k = 1.38 \cdot 10^{-23}\ \mathrm{J/K})$$
$$N = \text{Number of particles}$$
$$n = \text{Amount of substance}$$
$$R = \text{universal gas constante } (R = 8.3144621\ \mathrm{J/mol/K})$$
$$T = \text{absolute temperature (in Kelvin)}$$

As is often the case of physics, this is only an approximation of the real situation. It actually only applies exactly to an ideal gas in which the individual gas particles only collide with each other and otherwise do not exert any (long-range) forces on each other. In addition, the intrinsic volume of the gas particles should be very small compared to the actual gas volume. (Note: A gas law in which this idealization is not required is the so-called VAN DER WAALS EQUATION).
In this experiment, we will examine the three most important cases and conclusions of the gas law using air as an example:

a) BOYLE-MARIOTTE'S LAW:

$$pV = \text{konst.} \qquad \text{für} \quad T = \text{konst.} \tag{1.15}$$

b) GAY-LUSSAC'S LAW:

$$V \sim T \qquad \text{für} \quad p = \text{konst.} \tag{1.16}$$

c) AMONTONS'S LAW:

$$p \sim T \qquad \text{für} \quad V = \text{konst.} \tag{1.17}$$

1.3 Experiment

1.3.1 Equipment

part	quantity
Gas thermometer (inner diameter 2.7 mm)	1
Large glass tube	1
V-shaped tripod base, small	1
Tripod rod 47 cm, 12 mm Ø	1
Pipe collar with a clamp	2
Vacuum hand pump	1
Digital temperature sensor	1
Heater plate	1
Beaker, 250 ml	1

1.3.2 Experimental setup and procedure

Caution

- The gas thermometer and the large glass tube are very fragile.
  Always handle them with great care.
- Push the vacuum pump hose very carefully onto the gas thermometer.
  Too much pressure can easily break it.
- Only attach the glass parts to the clamps with minimal pressure.
- The water should only be heated in the beaker and on the hotplate. The water is then poured into the glass tube using the funnel. Put on the heat gloves to avoid burns from the hot glass. Make sure that you do not spill any water, the gloves are *not* waterproof.

Collecting the mercury beads

- Connect the hand pump and hold the gas thermometer with the hose connection facing downwards (see Fig. 1.2).
- Create maximum negative pressure Δp with the hand pump and collect the mercury in a drop in the bulge (a). (The pressure gauge shows the negative pressure Δp as a negative value).
- Bring any small mercury beads into the bulge (a) by tapping lightly against the glass tube. (A small mercury bead that remains attached to the melted end of the glass tube does not affect the measurement).

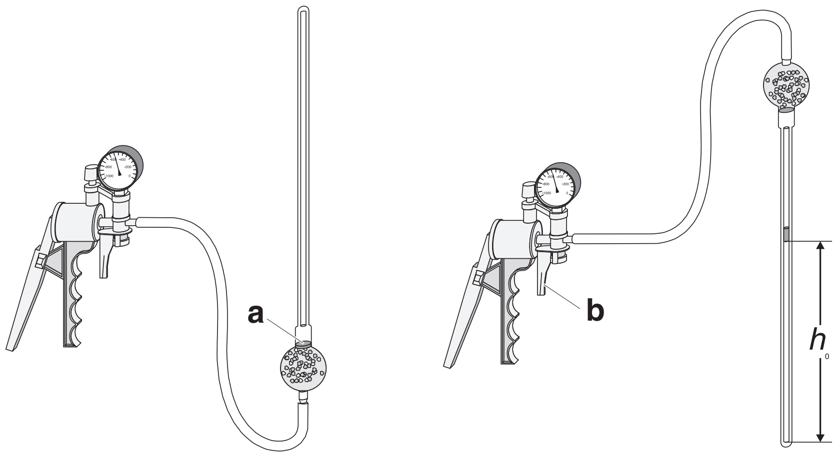

Abbildung 1.2: Collection of the mercury beads and adjustment of the initial volume V_0.

Boyle-Mariotte's law

- Collect the mercury beads.
- Slowly turn the gas thermometer to the operating position (hose connection upwards) and bring the mercury into the inlet of the glass tube.
- Slowly reduce the negative pressure Δp to 0 by carefully opening the ventilation valve (b) of the hand pump so that the mercury slowly moves downwards as a constricting plug.
- Mount the gas thermometer in the stand material as shown in Fig. 1.3. (If the mercury plug shatters due to excessive ventilation or due to shattering, collect the mercury again).
- Read the external pressure p_0 and room temperature T_0 at the weather station.
- Read the Höheight h_{Hg} of the mercury plug on the scale of the gas thermometer.
- Use the hand pump to create a vacuum Δp and gradually increase the pressure.
- For a total of 20 values, read off the Höheight h of the air column and note it down together with Δp.
- Repeat the measurement five times.

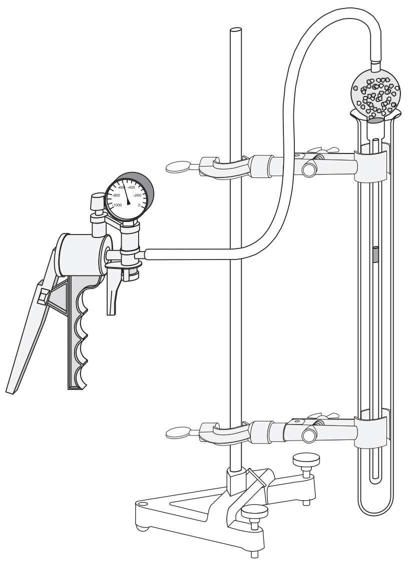

Abbildung 1.3: Experimental setup for investigating the pressure dependence of the gas volume at constant temperature.

Gay-Lussac's law

- Collect the mercury beads.
- Slowly turn the gas thermometer to the operating position (hose connection upwards) and bring the mercury into the inlet of the glass tube.
- Slowly open the ventilation valve of the hand pump ((b) in Fig. 1.2) to reduce the negative pressure Δp to 0 so that the mercury slowly slides down in one piece.
- Mount the gas thermometer in the stand material as shown in Fig. 1.4 and carefully remove the hose from the hand pump. (If the mercury plug bursts due to excessive ventilation or due to exhaustion, collect the mercury again).
- Carefully insert the temperature probe into the large glass vessel parallel to the gas thermometer and connect it to the digital thermometer.
- Heat about 200ml of water in the beaker on the hotplate to about 90°C.
- Fülle das Wasser vorsichtig in das grosse Glasgefäss. (Auf Grund der steigenden Temperatur steigt das Gasvolumen.)
- Carefully pour the water into the large glass jar. (The gas volume increases due to the rising temperature.)
- Observe the rise in temperature and wait until the temperature starts to fall again before reading the temperature and the height.
- For a total of 15 values, measure the temperature T between 85° Celsius and 35° Celsius and the height h of the enclosed gas volume of the gas thermometer as the heat bath cools down (duration approx. 50 minutes).
- Make a reasonable error estimate of the measured values.

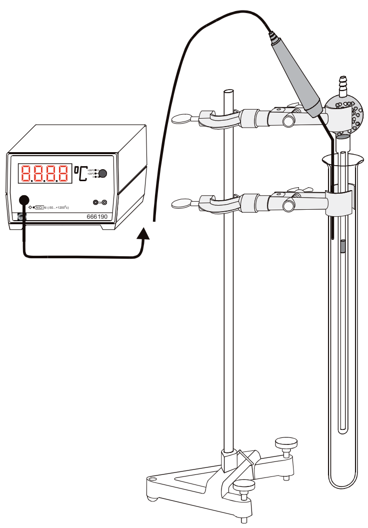

Abbildung 1.4: Experimental setup for investigating the temperature dependence of the gas volume at constant pressure.

Amontons' law

- Collect the mercury beads.
- Slowly turn the gas thermometer to the operating position (hose connection upwards) and bring the mercury into the inlet of the glass tube.
- Slowly open the ventilation valve of the hand pump ((b) in Fig. 1.2) to reduce the negative pressure Δp to 0 so that the mercury slowly slides down in one piece.
- Mount the gas thermometer in the stand material as shown in Fig. 1.5 (If the mercury plug cracks due to excessive ventilation or vibration, collect the mercury again).
- Carefully insert the temperature probe into the large glass vessel parallel to the gas thermometer and connect it to the digital thermometer.
- Heat about 200 ml of water in the beaker on the hotplate to about 90°C.
- Carefully pour the water into the large glass container. (The gas volume increases due to the rising temperature).
- Observe the rise in temperature and wait until the temperature starts to fall again before reading the temperature and height.
- Create a low negative pressure δ_p, which can be read on the scale of the hand pump (e.g. 60 mbar) and read off the height h_0 of the mercury plug.
- As the heat bath cools down, repeat the following steps for a total of 10 different temperatures: Determine the negative pressure Δp that must be generated by pumping with the hand pump until the mercury plug has reached its original height. Measure the temperature T between 85° Celsius and 35° Celsius and the negative pressure Δp for a total of 20 values (duration approx. 50 minutes).
- Make a reasonable error estimate of the measured values.

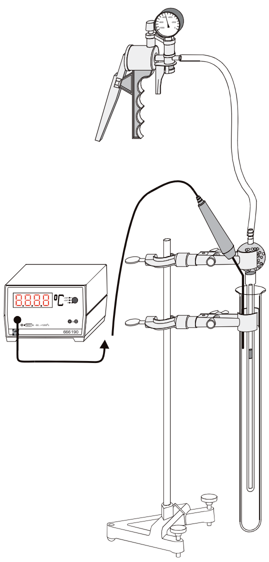

Abbildung 1.5: Experimental setup for investigating the pressure dependence of the gas volume at constant volume.

1.3.3 Tasks for Evaluation

Complete the following tasks and carry out a comprehensive error analysis in each case.

Boyle-Mariotte's law

- Divide the pressure range into 15 equally sized ranges and calculate the corresponding mean values and statistical errors from all measurement series.
- Calculate the pressure p from the measured values, which determines the volume of the enclosed air column.
- Investigate the validity of Boyle-Mariotte's law qualitatively and quantitatively.

Gay-Lussac's law

- Investigate the validity of Gay-Lussac's law qualitatively and quantitatively.
- Determine the absolute zero of temperature.

Amontons' law

- Investigate the validity of Amonton's law qualitatively and quantitatively.
- Determine the absolute zero of temperature.