Rutherford Scattering

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1.1 Preliminary Questions

- Describe the scattering experiment of Rutherford in your own words.
- In this experiment, a so-called discriminator is used. What is it and why is it necessary?
- Why is the experiment done in a vacuum chamber and not in air?
- How does an oscilloscope function? What are its general settings?

1.2 Theory

1.2.1 Development of the Atomic Model

The first model of the atom was from the Greek philosophers Leucippus (around 440 BC) and his student Democritus (460-370 BC). They believed that all matter is made of small, indivisible particles. These particles were called atoms after the Greek word for indivisible. Between these atoms, in their view, only empty space would be found. Therefore, all macroscopic bodies and their similar or different properties would, by their arrangement, be determined. However, throughout the Middle Ages and up to the beginning of the modern era, this idea was again discarded because it was too materialistic and also disagreed with the doctrine of creation by the church.

From about the 17-th century, the atomic model became more important again. Various chemists recognized by accurate weighing of the reactants and reaction products, that integer ratios resulted in their masses. These mass ratios were described by molecules from individual atoms.

Further evidence came from the development of the kinetic theory of gases by Rudolf Julius Clausius (1822-1888), James Clerk Maxwell (1831-1879), and Ludwig Boltzmann (1844-1906). They could attribute the macroscopic properties of gases such as pressure, temperature and specific to heat movement and bumps of gas atoms.

At the end of the 19-th century, the idea that the matter consists of atoms finally prevailed. Various experiments recorded a more accurate description of these atoms. Studies on the conductivity of different liquids, experiments with gas discharges and the discovery of radioactivity showed that in atoms, positive and negative charge carriers must be present. Thus, the atom was no longer "indivisible", but consisted of charged components. Mass spectrometry experiments showed that the positively charged components of the atom have a much larger mass than the negatively charged components. Also, thermodynamic experiments gave information about the atomic radius.

However, on one point, there was still uncertainty: how were the positive and negative charges distributed in the atom? The accepted model at the time was the so-called Raisin cake model, proposed by Joseph John Thomson (1856-1940). This model, also called Thomson’s Atomic Model, proposes that the positive and negative charges are evenly distributed in the atom.

1.2.2 Scattering Experiments

The question of the charge distribution in the atom could finally be clarified in 1911 by Ernest Rutherford (1871-1937) and his staff. They took advantage of the then novel technique of scattering experiments, in which a beam of incident particles of type A interacts with a target made of another type of particles B. Then, we determine how many particles of type A are
scattered within the solid angle $d\Omega$. From such experiments, one can draw conclusions about the interaction potential $V(r)$ between the particles A and B.

The most important concept in scattering experiments is the so-called cross-section, which is defined as follows:

$$\sigma_{\text{tot}} = \frac{\text{number of scattering events per unit time}}{\text{number of incoming particles per unit time} \times \text{number of scattering centers per unit area}}.$$  

The cross-section has units of area, but it is neither the physical dimension of the target, nor the surface of the individual scattering centers. Instead, it is a measure of the probability of interaction between incident particles and the target. The cross-section associated with the particles scattered within the solid angle $d\Omega$ is the so-called differential cross section $d\sigma/d\Omega$.

Considering the characteristics of the incoming beam and the target, $\sigma_{\text{tot}}$ can be specified as follows:

$$\sigma_{\text{tot}}(E) = \frac{N_{\text{tot}}}{F_{\text{inc}} A n t}, \quad (1.1)$$

where $N_{\text{tot}}$ is the number of scattered particles per second ($s^{-1}$), $F$ is the flux of incident particles ($m^{-2}s^{-1}$), $A$ is the area of the target or the area covered by the incident beam ($m^2$), $n$ is the atomic concentration of the target ($m^{-3}$) and $t$ is the thickness of the target ($m$). The cross-section is generally dependent on the energy $E$ of the incoming particles. It is common to find the cross-section in barn, where: $1$ barn $= 10^{-28}$ m$^2$. If $N_S$ is the number of particles scattered per second within the solid angle $d\Omega$, then the differential cross section is defined as follows:

$$\frac{d\sigma}{d\Omega}(E, \Omega) = \frac{1}{F_{\text{inc}} A n t} \frac{dN_S}{d\Omega}, \quad (1.2)$$

with $N_{\text{diff}}$ given by:

$$N_S(E, \Omega) = F_{\text{inc}} A n t \frac{d\sigma}{d\Omega}(E, \Omega) \quad (1.3)$$

The relationship between total and differential cross section is given by equation (1.4). The second equality is obtained by using the definition of differential solid angle in spherical units ($d\Omega = \sin \theta d\theta d\phi$).

$$\sigma_{\text{tot}}(E) = \int \left( \frac{d\sigma}{d\Omega} \right) d\Omega = \int_0^{2\pi} d\phi \int_0^\pi \frac{d\sigma}{d\Omega} \sin \theta d\theta, \quad (1.4)$$

### 1.2.3 The Rutherford Experiment

Ernest Rutherford, Hans Geiger and Ernest Marsden investigated the charge distribution of the atoms with scattering experiments as described above. Inside an evacuated chamber, they shot a beam of $\alpha$-particles ($^4_2\text{He}^{2+}$) into a very thin gold foil. Using a fluorescent screen, they measured the count rate at different angles in respect to the incident beam. The influence of electrons in the scattering can be neglected due to their small mass. Due to the Thomson’s atomic model, Rutherford expected that the incident particles would exhibit only very small scattering angles. Indeed, the scattering experiments showed that most of the incident $\alpha$-particles easily penetrated the film. However, the deflection of the incident particles at large angles is a strong evidence of the existence of protons confined in a small nucleus inside the atom, whereas the electrons would orbit around the nucleus.
The differential cross section for the scattering of charged particles with incident energy $E_{\text{kin}}$ is the so-called Rutherford cross section and it can be shown that it is given by (for instance refer to Bransden & Joachain (2003)):

$$
\left. \frac{d\sigma}{d\Omega} \right|_{\text{Rutherford}} = \left( \frac{q Q}{4 \pi \epsilon_0} \right)^2 \left( \frac{1}{4 E_{\text{kin}}} \right)^2 \frac{1}{\sin^4 \left( \frac{\theta}{2} \right)},
$$

(1.5)

where $Q$ is the charge of the nuclei inside the target (C), $q$ is the charge of the incident particles (C), $E_{\text{kin}}$ is the kinetic energy of the incoming particles (J), $\theta$ is the angle between the incident beam and scattering direction, and $\epsilon_0$ is the vacuum permittivity ($8.854 \times 10^{-12}$ F m$^{-1}$). Equation (1.5) is obtained for a radially symmetric configuration (Figure 1.2), in which the solid angle $d\Omega$ corresponds to the area delimited by $[\theta, \theta + d\theta]$ (i.e., the scattering is independent of $\phi$).

The number of $\alpha$-particles $N_S(\theta)$ with an incoming energy $E_\alpha$ that scatter off a material with atomic number $Z$ is calculated using Eqs. (1.3) and (1.5):

$$
N_S(\theta) = N_{\text{inc}} n t \frac{Z^2 e^4}{(8\pi\epsilon_0 E_\alpha)^2 \sin^4 \left( \frac{\theta}{2} \right)},
$$

(1.6)

where $N_{\text{inc}} = F_{\text{inc}} A$ is the flow of the incoming $\alpha$-particles (s$^{-1}$), $E_\alpha$ is their kinetic energy and $e$ is the elementary charge ($1.602 \times 10^{-19}$ C).
1.3 Experiment

The apparatus you will work with is presented in figure 1.3, where the main components are identified. In these experiments you will use Am-241 as a-source, which decays with an half-life of about 432.2 years and the average energy carried out by the α-particles is around 5.48 MeV. The source is placed inside the scattering chamber, in front of a target of gold or aluminium. On the back of the target, a particle detector is used to count the number of particles, as presented in figure 1.4. The discriminator and the oscilloscope are used to help filter out the false events from the true events (i.e., those from the α-decay) as displayed in figure 1.5.

By varying the angle θ between the source/target and the detector, the number of scattered
particles is expected to follow the dependency given by equation (1.6), which will be verified during these experiments. Two targets of gold and aluminium are used and the main task of this work is to experimentally obtain the thickness of the Au foil.

Figure 1.4: Scattering chamber: (1) sample holder, (2) support for the foil, (3) Foil, (4) slit, (5) small hinged arm, (6) detector.

Figure 1.5: Example of the signals obtained with the oscilloscope depending on the threshold set with the CURSOR/TRIGGER functions.
1.3.1 Equipment

<table>
<thead>
<tr>
<th>Component</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scattering chamber</td>
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</tr>
<tr>
<td>Vacuum pump with pressure sensors</td>
<td>1</td>
</tr>
<tr>
<td>Gold foil with slit</td>
<td>1</td>
</tr>
<tr>
<td>Aluminum foil with slit</td>
<td>1</td>
</tr>
<tr>
<td>Am-241 source</td>
<td>1</td>
</tr>
<tr>
<td>Discriminator preamp</td>
<td>1</td>
</tr>
<tr>
<td>Counter</td>
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</tr>
<tr>
<td>Oscilloscope</td>
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</tr>
<tr>
<td>BNC-to jack cable</td>
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<td>BNC short cable</td>
<td>1</td>
</tr>
<tr>
<td>BNC long cable</td>
<td>2</td>
</tr>
<tr>
<td>BNC T-Piece</td>
<td>1</td>
</tr>
<tr>
<td>Blanket</td>
<td>1</td>
</tr>
</tbody>
</table>

1.3.2 Considerations for before and after the experiments

Before starting the experiments, follow the steps below so that you can ensure the highest quality of your measurements:

1. open the scattering chamber and place the Am-241 source in the sample holder (see Figure 1.4, (1)) and the gold foil in the corresponding holder (see Figure 1.4, (2)). The slit has a notch on one side that allows for alignment. The side with the slot-shaped aperture should point at the source

2. the detector has an aperture slit, which should be aligned to that of the foil

3. close the chamber and turn the small hinged arm (see Figure 1.4, (5)) to the wall so that it does not interfere with the measurements

4. open the valve at the exit of the chamber and close the vent valve on the vacuum pump

5. start the pump and wait until the pressure has fallen to 5 mbar or less. Let the pump run throughout the experiment

6. connect the detector to the discriminator pre-amplifier using the short BNC. Connect also the signal output of the pre-amplifier to the oscilloscope

7. align the source/foil/detector so that \( \theta = 0^\circ \) and set the discriminator threshold voltage to 0 V. What do you observe with the counter? By decreasing the voltage the number of counts decreases. Why?

8. using the CURSOR and TRIGGER functions of the oscilloscope, set the limit below which the signals are only due to the \( \alpha \)-particles scattered by the foil (refer to figure 1.5). The discriminator threshold voltage should be adjusted accordingly, so that the number of counts matches the signals at the oscilloscope

The measurements can now be performed following the procedure presented in section 1.3.3. After completing the measurements, it is essential that you do the following steps to correctly vent the chamber:

1. close the outlet valve of the Rutherford scattering chamber
2. switch off the vacuum pump
3. open the vent valve on the pump. Make sure that you hear a hiss
4. open the valve of the scattering chamber slowly until a hiss is audible
5. wait until the hissing has stopped

1.3.3 Procedure

Measure the count rate every $\pm 5^\circ$ up to $\pm 30^\circ$ with the counter GATE set to 100 s. During the experiments follow the procedure below:

1. place the large arm to the desired position.
2. cover the scattering chamber with the blanket.
3. zero the counter and start it. Watch the signals on the oscilloscope. Evaluate if you identify faulty signals. In the case of no faulty signals, write down the number of hits. If faulty signals occur, the measurement must be discarded and a new set of measurements performed
4. repeat the 100 s-window measurements so that, for the Au foil, the total number of counts reaches at least a minimum total of 100 hits for $\theta = \pm 5 - 20^\circ$. For $\theta = \pm 25^\circ$ the total number of counts should be at least 50, and with $\theta = \pm 30^\circ$ you should obtain at least 10 counts
5. when the Au-target experiments are finished, vent the scattering chamber (following the procedure presented in section 1.3.2) and insert the Al foil. Due to the lower scattering yield, lower counts are expected with aluminium. Therefore, consider the following counts thresholds: 100 for $\theta = \pm 5^\circ$, 50 for $\theta = \pm 10^\circ$, 15 for $\theta = \pm 15^\circ$ and 10 for the remaining angles.

1.3.4 Tasks for Evaluation

1. compute for gold and aluminum foil, the direct count rate $N_d$ for each angle
2. the determined $N_d$ rates are a function of the angle $\theta$ and calculated in the plane. To be able to compare them with the Rutherford scattering formula (Eq. 1.6), the rates must be corrected. Using equation (1.4), the corrected count rate is written as follows:

$$N_S = 2\pi \sin \theta N_d$$

(1.7)

3. plot $N_S(\theta)$ as a function of $\theta$. The data points must be accompanied by error bars corresponding to the uncertainties of the measured counts, which are calculated using the Poisson statistics: $\Delta N = \sqrt{N}$, where $N$ is the number of counts. The statistical uncertainty in the measurement of time and angle measurement can be neglected. Choose a logarithmic representation of the axis with the rates and interpret the plot
4. fit the following function to your measurements:

$$f(\theta) = \frac{a}{\sin^4 \left( \frac{\theta - b}{2} \right)}$$

(1.8)

Give the values of $a$ and $b$. Which statement can you derive from them?
5. from the ratio of the rates $N_{S, \text{Au}} / N_{S, \text{Al}}$, one can, with the help of equation (1.6), estimate the thickness of the gold film (expected to be $2 \mu m$), knowing that $t_{\text{Al}} = 8 \mu m$. Furthermore, assume that the atomic concentration of both films, the number of particles and their energy are equal. For each angle $\theta$, calculate the thickness of the gold film. Determine the average value with its statistical uncertainty, and compare it with the expected thickness.

1.4 Literature