IE4

Modul Electricity

Specific Charge of the Electron

Goal of this experiment is to determine the specific charge of the electron by measuring the deviation of an electron beam caused by a homogeneous magnetic field. The homogeneous magnetic field is generated by a special arrangement of coils. This arrangement is called HELMHOLTZ coils.

Versuch IE4 - Specific Charge of the Electron

Goal of this experiment is to determine the specific charge of the electron by measuring the deviation of an electron beam caused by a homogeneous magnetic field. The homogeneous magnetic field is generated by a special arrangement of coils. This arrangement is called HELMHOLTZ coils.

1.1 Preliminary Questions

In principal all students, performing this experiment, should be familiar with the basics of electro- and magneto statics. The following list of characteristical questions respective subjects are typical for electro-/magneto statics and relevant for this experiment. The mastering of these questions/subjects is the minimal requirement to pass this experiment. However it is recommended that the students earn a deeper understanding by studying the stated literature, if you're not familiar with these subjects.

- Electric field origin in electric charge, but how are magnetic fields generated?
- How does the magnetic field of a current-carrying conductor look like? How can you calculated it?
- How can you build a coil from a current-carrying conductor as used in this experiment?
- How is a charged particle deflected in a magnetic field, if it moves parallel/perpendicular to the field lines?
- Why is the natural constant μ_0 ? (Hint: How is this constant connected to the speed of light c)
- How is the weighted average and the used weights defined?
- Cite a reasonable value for the systematic error of the electron path radius.
- Derive the formula to calculate the specific charge of the electron with use of the information given in the section "'Theory"'.
- Search for the literature value of the specific charge of the electron.
- In this experiment, electric and magnetic fields are used to direct electrons. Name other experiments that use the same principles.

1.2 Theory

The force acting on a moving charge in a electric or/and magnetic field is called LORENTZ force. One has:

$$\vec{F} = q \cdot \vec{v} \times \vec{B} + q \cdot \vec{E} \tag{1.1}$$

Here q is the charge of the electron, $\vec{B}$ is the magnetic field and $\vec{E}$ is the electric field. Especially in older textbooks only the term $q \cdot \vec{v} \times \vec{B}$ is described as LORENTZ force and the second term is referred to as Coulomb force. Imagine now a Cartesian coordinate system in which the z-axis is parallel to the homogeneous magnetic field $\vec{B}$. We consider an electron which moves parallel to the x-axis. It should be highlighted that only constant velocities $\vec{v}$ are regarded in the following, meaning that the motion is uniform. In the homogeneous magnetic field the LORENTZ force acts on this electron. The force is perpendicular to $\vec{v}$ as well as to $\vec{B}$ and therefore points into the y-direction. Consequently the force does not perform any work on the electron. The force only causes the electron to move in a circular motion. We can now ask the question, whether one can deduce the charge q of the electron solely from this fact? We consider the simple case that $\vec{v}$ and $\vec{B}$ are perpendicular to one another. Because the electron performs a circular motion under the influence of the LORENTZ force, the force has to be equal to the centripetal force $\vec{Z}$ of a circular orbit. If the mass of electron is denoted by m, we get the following equation

$$\begin{aligned} \left|\vec{Z}\right| &= \left|\vec{F}\right| \\ \frac{m \cdot |\vec{v}|^2}{r} &= q \cdot |\vec{v}| \cdot |\vec{B}| \\ R &= \frac{m \cdot |\vec{v}|}{q \cdot |\vec{B}|}, \end{aligned}$$

where r is the radius of the circular orbit. Now we still need the velocity $\vec{v}$ of the electron. In our case, it is enough to know its absolute value $v = |\vec{v}|$. The electrons get thermally ejected from a hot cathode and then accelerated with the help of an electric field $\vec{E}$ with voltage U in a WEHNELT cylinder. This means: we increase the kinetic energy of the electrons in the metal thread by heating until the electrons pass out of the wire. Afterwards we apply an electric field. The electrons are accelerated along this field. So we can use the law of energy conservation and therefore equate the kinetic energy of the electrons with the electric energy of the electrons. Thereby we neglect relativistic effects, but in our case this is not a problem, because the velocities are very small compared to the speed of light.

$$\begin{aligned} \frac{1}{2} \cdot m \cdot v^2 &= q \cdot U \\ v &= \sqrt{\frac{2 \cdot q \cdot U}{m}}. \end{aligned}$$

Finally we need the magnetic field of the HELMHOLTZ coils (see fig. 1.1). It is a special geometry of conducting coils which has the purpose to generate a nearly homogeneous field in between the coils. The derivation of the magnetic field of such coils is given in the appendix.

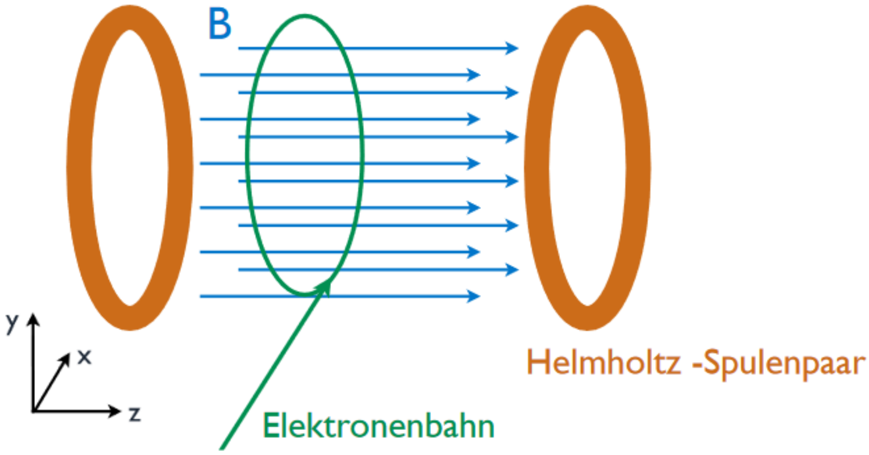

Figure 1.1: Assembly of the HELMHOLTZ coils. The radius R of the coils corresponds to the distance between them. The magnetic field inside the coils is approximatly homogenious.

Here we simple use the resulting equation for the magnetic field B:

$$B = \left(\frac{4}{5}\right)^{\frac{3}{2}} \cdot \frac{\mu_0 \cdot N \cdot I}{R},$$

where μ_0 is the vacuum permeability, N the number of coil windings, I the electric current and R the radius of the coils.
By inserting the formulas for B and v we can now conclude on the quotient q/m, which is described as specific charge of the electron. From this experiment only the ratio q/m of an electron can be measured and not the electron charge q (or the mass m) by itself. Certainly the mass of an electron can be measured with experiments from particle physics and is well known today, but out of didactic reasons we want to follow the historic model and determine the specific charge of the electron in this experiment.

1.3 Experiment

You will find the experiment in an unwired condition. Your first experimental task is to wire the experiment correctly and then get it checked by the assistant. Only by confirmation of the assistant you can go on with the experiment. Check if all the needed material is available (see list below).

1.3.1 Supplies

Component	Quantity
Evacuated glass bulb with WEHNELT cylinder and hot cathode	1
Ruler	1
Power supply with 2 multimeters	1
Pair of HELMHOLTZ coils ($R = 15\ cm, N = 125$)	1

1.3.2 Experimental Procedure

To allow a long lifetime of the equipment the following rules should be considered:

- Initial operation:
    1. Turn all buttons to the left (**OFF**).
    2. Turn on the voltage supply.
    3. Heating current is now **ON** (1 A). Wait approximately 5 min.
    4. Anode voltage **ON**.
    5. Turn the button 'V' of the power supply all the way to the right an turn it on. It should be in 'constant current mode'.
- During operation:
    1. By varying the heater current and the focussing the concentration of the electron beam can be optimized.
    2. When a beam becomes visible, the magnets current can be adjusted until it forms a circular path.
    3. Perform the following measurements:
        - Set the voltage U constant (in between 200-300V) and measure the radius r as a function of the electric current I, resp. the magnetic field $\vec{B}$ (at least 15 data points!)

    - Set the electric current I, resp. the magnetic field $\vec{B}$, constant (in between 1.5-2 A) and measure the radius r as a function of the voltage U (at least 15 data points!)

- Shut-down:
  1. Turn all buttons to the left (**OFF**).
  2. Turn off the voltage supply..
  3. Power supply **OFF**.
  4. Unwire everything and bring the experiment in its initial state.
  5. After the experiment the measurement data and experiment workplace must be shown to the assistant.

The experiment consists of an evacuated glass bulb which is mounted in between the HELMHOLTZ coils. In this glass bulb there is a WEHNELT cylinder with a hot cathode. This cathode emits thermal electrons that are accelerated by the cylinder with variable voltage.

Now the magnetic field forces the electrons on a circular orbit. To see the circular orbit, the glass bulb is filled with a small amount of noble gas. If the electrons hit a gas atom, they transfer an amount of kinetic energy to the atom and the atom gets to an excited state. The atoms then drop back to their ground state by emitting a photon. The photons generated by this interaction make the circular orbit of the electrons visible.

1.3.3 Tasks for Evaluation

1. Investigate the orbit radius r in dependence of the acceleration voltage U and the magnetic field $\vec{B}$.

2. Determine the specific charge of the electron q/m from your measurements. Naturally you need to do a full error analysis. In the appendix you find the calibration measurement of the coils. From this data you can calculate the magnetic field B for every measured current I.

3. Is it reasonable to use the weighted average in this experiment? If yes, calculate the weighted average. Compare your final result with the literature value. Is the literature value within the error bars of your final result?

4. Which are the main measurement uncertainties in this experiment? How could they be optimized?

5. Plot your data points in diagrams ($r(U)$ and $r(I)$) and compare their functional dependencies with the theoretical expectations.

Literature

- Demtröder Band 2 - *Elektrizität und Optik*, 6.Auflage: Kapitel 2, Abschnitt 3.2 und 3.3

- *Gerthsen Physik*, 22. Auflage oder neuer: Kapitel 6, Elektromagnetismus, Abschnitte 6.1, 6.8 und 6.9

- Nolting Band 3 - *Elektrodynamik*, 8. Auflage: Kapitel 3, Abschnitt 3.2.1

Appendix

A.1 The magnetic field of HELMHOLTZ coils

HELMHOLTZ coils are a pair of coils which are arranged in a way that in between the coils the magnetic field is nearly homogeneous. Thereby the same current is flowing through each coil in the same direction. The coils have a radius R and a number of windings N. They are also separated by the distance R. To calculate the magnetic field $\vec{B}$ of this arrangement, first we need to calculate the magnetic field of a flat circular coil (see fig. A.2). Here we need the BIOT-SAVART law. This law can be described by the following formula:

$$\vec{B} = \frac{\mu_0 \cdot I}{4\pi} \int_L \frac{d\vec{s} \times \vec{r}}{r^3}.$$

Here $d\vec{s}$ is a piece of the conductor, $\vec{r}$ the position vector, which connects the piece of conductor with the reference point, I is the electric current flowing through the conductor and μ_0 describes the vacuum permeability. The integration is done along the conductor.
In our case the field along the coil axis is of interest and therefore $\vec{r}$ is constant. Furthermore it is clear that along the coil axis all components perpendicular to it are cancelling each other. We only have components along the coil axis.
Let φ be the angle between axis and magnetic field. Then the integral can be written as:

$$|\vec{B}| = \frac{\mu_0 \cdot I}{4\pi} \int_L \frac{ds \cdot r}{r^3} \cdot \sin\varphi,$$

$$B = \frac{\mu_0 \cdot I}{2} \cdot \frac{R^2}{r^3} \cdot N.$$

Here we replaced the vectors by their absolute values, $\sin\varphi = R/r$ and $\int ds = 2\pi \cdot R$.
Now we only need to apply this result to the coil pair. We calculate the magnetic field in the center of the coil. Here the distance r is given by:

$$r = \sqrt{R^2 + R^2/4} = \sqrt{\frac{5}{4}} \cdot R.$$

Used in the above equation for B and multiplied by two (because we have two coils) we get the magnetic field for a HELMHOLTZ coil pair:

$$\boxed{B_H = \left(\frac{4}{5}\right)^{\frac{3}{2}} \cdot \frac{\mu_0 \cdot N \cdot I}{R}}$$

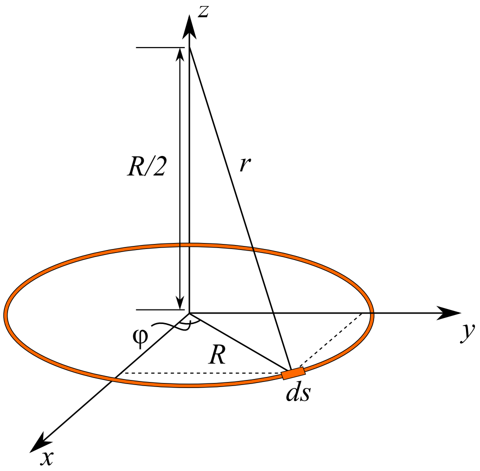

Figure A.2: Sketch to illustrate the above derivation. Hereby *ds* denotes the line element along the integration, the radius of the coil is R and φ is the angle. To calculate the magnetic field of the conductor loop cylindrical coordinates are used, which means it must be integrated over z, R and φ.

A.2 Figure of the Experimental Setup and the Wiring

The following illustrations should help to build the experimental set up and point out how to plug in the wires correctly. Please have a look at figures A.3 and A.4.

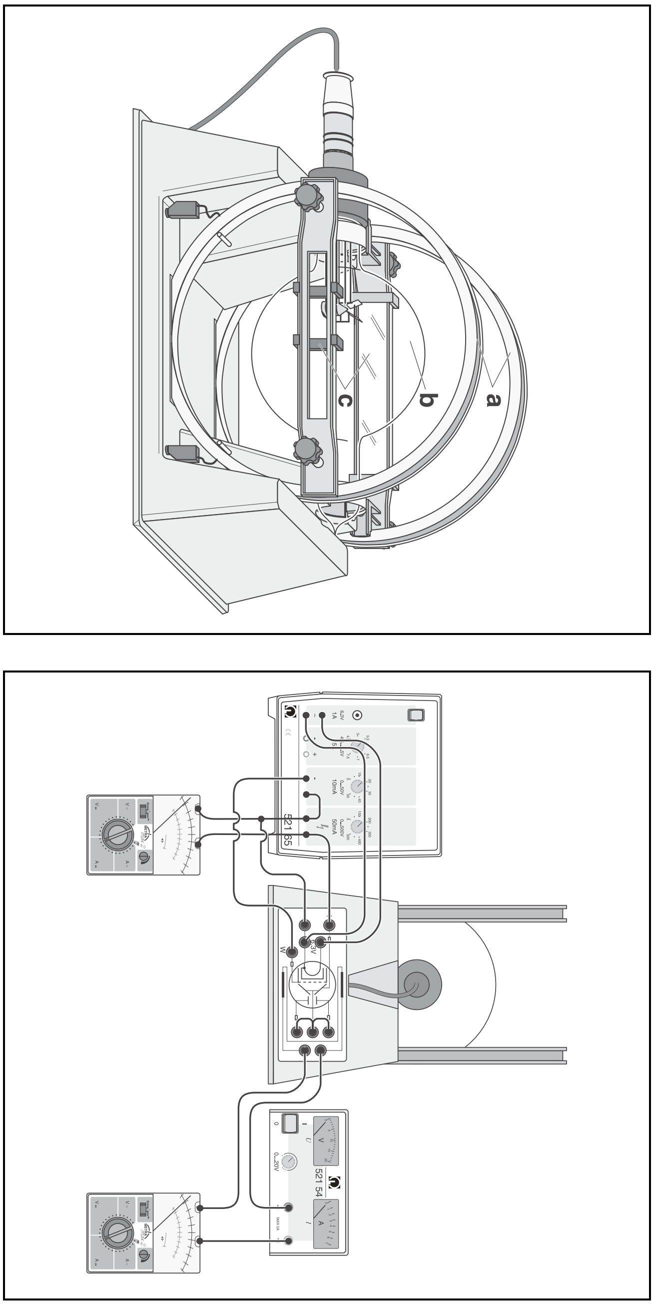

Figure A.3: Experimental setup: (a) are the HELMHOLTZ coils, (b) is the electron beam, and (c) is the ruler to read the radius of the beam. The drawing at the bottom shows the correct wiring.

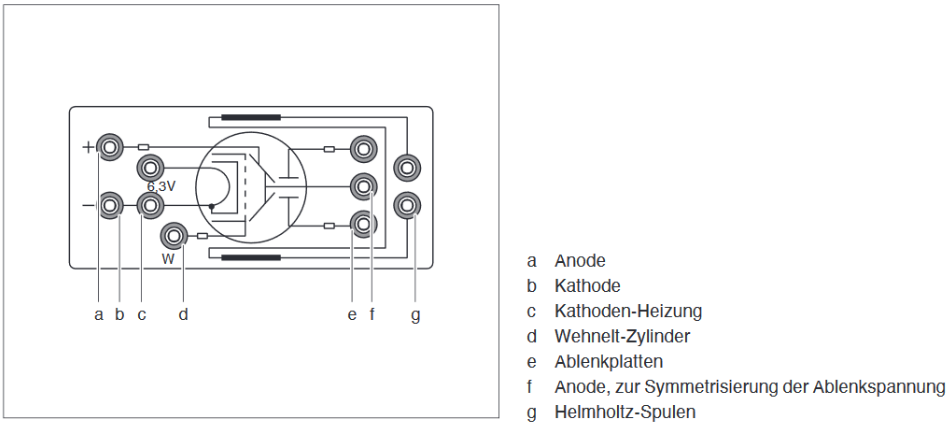

Figure A.4: The anode is marked with a + symbol, the cathode with a − symbol. The cathode heating is (c), the cylinder is (d) and (e) is the baffle. In (g) the HELMHOLTZ coils can be powered.

A.3 Calibration

The following plot shows the relationship between the current I and the magnetic field $\vec{B}$. It includes a linear fit. With the help of these fit parameters, the magnetic field inside HELMHOLTZ coils can be calculated.

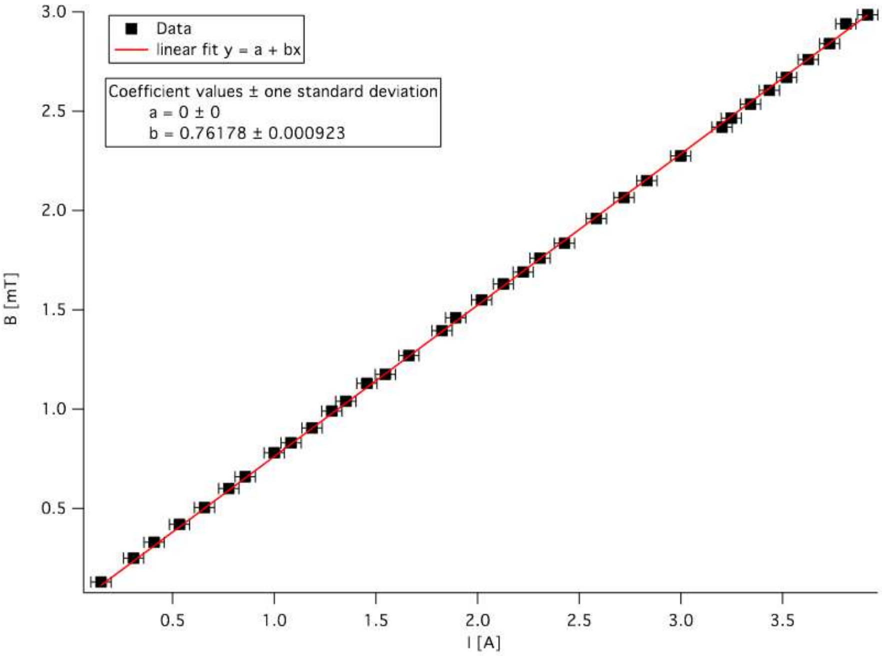

Figure A.5: Measurement data and linear fit of the calibration of the magnetic field from the HELMHOLTZ coils. With the slope of the linear fit the magnetic field can be calculated for each measured current.